EVALUATION OF ELASTIC PROPERTIES OF RANDOMLY ORIENTED FILAMENTARY COMPOSITES BY FINITE ELEMENT METHOD

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মার স্মৃতিতে
বাবার উদ্দেশ্যে
কমল

IN MEMORY OF MY MOTHER
WITH REGARDS TO MY FATHER

KAMAL

CERTIFICATE

Certified that the work 'EVALUATION OF ELASTIC PROPERTIES OF RANDOMLY ORIENTED FILAMENTARY COMPOSITES BY FINITE ELEMENT METHOD' has been carried out under my supervision and has not been submitted elsewhere for a degree.

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ABSTRACT

Randomly oriented filamentary composites form an important group of materials. They find wide applications in both common and advanced engineering applications. But unfortunately only a limited investigations have been made in this area. A computer simulation technique using finite elements has been described here for the micromechanistic study of this type of materials to describe their in-plane elastic properties. A flat rectangular model with rectangular cross-section under tensile loading on the two ends is studied. The fibers have been randomly oriented using a technique based on random number generation.

The technique makes it possible to get the detailed stress distribution in the random structure which is necessary for the study of failure mechanics. It is found from the micro-mechanistic study that the maximum stress concentration factor in the random structure is much higher than that for regularly spaced fibers under identical conditions. However, the maximum shear stress seems to be of the same order. For the simulated random structure, an interesting in-plane bending effect is found which disappears slowly as the fibers tend to align in a particular direction. This needs thorough investigations, both theoretically and experimentally.

The macroscopic responses of randomly oriented filamentary composites have also been studied for the influence

of biasness, residual compressive stress and volume fraction. Residual compressive stress has almost no effect on the modulus (Young's or Shear). The increase in volume fraction with increased cross-sectional area of the reinforcements has little effect on the modulus for random structure. It slightly increases with increased volume fraction. But the biasness has great influence on the modului of the composites. They rapidly increase as the fibers align more and more in a particular direction.

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NOMENCLATURE

E	Young's modulus in psi
G	Shear modulus in psi
$\sigma_{x,y}$	Normal stress in x or y direction in psi
σ_{xy}	Shear stress in xy plane in psi
$\{\sigma\}$	Stress vector namely σ_x, σ_y and σ_{xy}
u, v	Displacement along x or y direction in inches
$\{u\}$	Displacement vector namely u and v
q	Nodal displacements in inches
$\{q\}$	Nodal displacement vector
$\{S\}$	Surface force vector in psi
$\{\varepsilon\}$	Strain vector, normal and shear strains
dV	Volume integral
V	Volume of the element
dA	Surface integral
A_2	Portion of the elemental area where displacement is prescribed
Π_c	Complimentary energy functional
U	Strain energy
$[k]$	Stiffness matrix of the element
Π_p	Potential energy functional
A_1	Portion of the elemental area where surface traction is prescribed
$\{Q\}$	Nodal load vector

$\bar{T}$	Prescribed surface traction in psi
$E_{f,m}$	Modulus of fiber reinforcement or the matrix
V_f	Fiber volume fraction
ν	Poisson's ratio
$\nu_{f,m}$	Poisson's ratio of fiber reinforcement or the matrix
$[K]$	Overall or Master Stiffness matrix of the structure

Bar indicates average value.

CHAPTER I

INTRODUCTION

1.1 Preamble:

Making composites is an old art but new science. Early civilisations are to be explored to find the genesis of composite As in many other walks of life, even modern science wonders over the artistic treasures made with older composites. Typical examples may be cited as[1] straw-reinforced brick (early Isralite civilisation), Incas' platinum dispersed gold and silver ornament (early American civilisation), Damascus (18^{th} Century) and Japane (16^{th} Century) swords, Indian flint lock gun (17^{th} Century). But arts alone cannot survive unless they are supplemented by science As the aerospace science and technology started acquiring momentu there was an acute shortage of material to meet the new needs. A a result of the search of new materials, many composites were discovered and the search is still going on. Since Sixties a conscious and deliberate applications of the basic philosophies of composites with the different materials have made the progress of this group of materials very fast. At the moment composites are extensively used on a commercial basis even for domestic purposes. They are being widely used in industries like aircraft and missiles, architecture and construction, transportation, machine tools, agriculture, consumer products and appliances - to name a few.[2]

1.2 The Perspective:

The importance of composite material is well established. Here, it may be of interest to review it from the point of nationa interest. It has been rightly pointed out in the Report on the Science and Technology Plan of NCST group on special materials[3] that the most widely applied composite, namely the Fibrous or Filamentary Composite, has a number of attractive features in the perspective of our socioeconomic needs. Traditional craftsmanship existing in the country can be economically and intelligently used for meeting many of the country's needs without going for much of sophistication. The report points out: "FRP (Fiber Reinforced Plastics) technology has an employment potential at both skilled labour and scientific development levels. The primary market will be local but export possibilities must surely arise later." Typical uses have been cited in agriculture, public health, housin and transport apart from other advanced applications in different industries. The major organisations involved in the development a applications of fibrous composites include V.S.S.C., N.A.L., C.G.C.R.I., B.A.R.C. etc. apart from the advanced educational institutes like I.I.T.s and I.I.Sc. In terms of money about 3.6 crores of rupees (about one fourth of total budget, Rs 15.15 crores on special materials) has been estimated for the development and application of the materials and Rs 10 lacs for basic research for 5 years and $ 3,26,000 from U.N.D.P. for development of newer fibers.

1.3 The Problem:

Under the perspective it is decided to synthesise the national need and latest technology available in the country. Finally, the problem is chosen from the area of filamentary or fibrous composites which apart from advanced technological applications in defence and aerospace industries, will find equal use in more common industries and areas with added scientific confidence.

A computer simulation technique has been developed to evaluate the in-plane elastic properties of filamentary composites where short, discontinuous fibers are randomly oriented. The analysis is restricted to plane stress problems within the elastic region. Finite Element Method is used to measure the structural response of the composite system.

The three compelling reasons for the selection of the specific problem are:

(1) To supply the 'intrinsic' material properties of filamentary composites as the starting point of a reliable optimal design for advanced space and military applications,

(2) To supply manufacturer and/or designer a tool to select the starting materials for their structural components of more common industries, and

(3) To help the fracture mechanics man with detailed stress distributions within the structure for analysis for a safer design.

A master program has been developed for analysing filamentary composites with upto 12 phases and their arbitrary distributi

The program can be used for continuous reinforcements also without any difficulty. In actual experiments, the possible combinations of the matrix and reinforcement are literally thousands and one set of experimental results are not, in general, applicable for the second set. Considering the time necessary for making the experiment and the cost involved, it is rarely used as a starting point of any design. On the contrary, the computer simulation technique established here, serves as a very effective tool for designing the desired structural properties from a large number of possible combinations of materials. Also, since this method is essentially a micromechanics study, it has its inherent advantages over other analytical methods which are essentially macromechanics conclusions.

CHAPTER II

LITERATURE SURVEY

2.1 Fiber Reinforced Composites:

Broutman[4] defines the composite materials as the man-made group of materials obtained by three-dimensionally mixing two or more chemically distinct materials which gives the properties unattainable by the individual constituents acting alone and there is no significant chemical reaction among the constituents. The base material in which the other materials are dispersed is known as the matrix and the dispersed phase(s) is (are) known as reinforcement(s). Depending upon the aspect ratio (length by diameter) of the reinforcement the composites are categorised. The fibrous or fiber reinforced or filamentary composites are those for which the aspect ratio is much larger than unity. Depending upon use, the fibers are made continuous or discontinuous in filamentary composites.

To fabricate filamentary composites the strong fibers, continuous or discontinuous, are randomly or preferentially aligned in the matrix while the latter is in molten state and the whole system is then cured under specific temperature and pressure. Special care is taken to drive away the air bubbles from the end product. Apart from different varieties of plastics, the metals are also occasionally used as the matrix material. The choice of fibers can be glass, boron or other advanced materials, metallic or

inorganic or natural fibers. Consequently the choice of filamentary or fibrous composites is almost unlimited.

In any filamentary composite the load is mostly taken up by the fibers. The matrix takes only a small part, if anything at all, of the total load. The main role of the matrix is to bind the fibers together and add stiffness and rigidity in transverse directions. Also it protects the fibers from environmental and frictional degradations. For discontinuous filamentary composites, it is the matrix through which the fibers pick up the load, they carry.

The host of problems the designer of filamentary composites needs to solve include complex microstructure, inhomogeneous distribution of different phases, various shapes and sizes of the reinforcements, their alignment and continuity, little understood interface, possible existence of voids and presence of residual stresses introduced during curing. This battery of problems are good enough to force any designer to make a series of assumptions in solving even the 'simplest' problem in filamentary composite. Now, in addition, if the fibers are assumed to be randomly distributed in the plane of composites, it becomes almost the last straw on the 'designer's' back! However, these host of problems never mean that no satisfactory solutions for composite structures exist. It only means that the designer must have the foresight to appreciate the relative importance of the simplifications he is making.

The final test of any structural design is the experiment. But the choice for filamentary composites being numerous, the experimental technique often becomes an uneconomic design procedure. Consequently, a number of investigators have come up with their approximate theories based on different assumptions. Though there are a number of analyses available for regularly spaced fibers,[5-7] the number is very few for randomly distributed filamentary composites. Cox[8] investigated the effect of orientations of short discontinuous fibers on filamentary materials and showed that for planar matrix all possible types of elastic behaviour can be predicted by composition of four approximate sets of parallel fibers. Without any consideration of the matrix material, he concluded that the isotropic properties for randomly oriented filamentary composites are given by

$$\bar{E} = E_{11}/3 \quad \text{and} \quad \bar{G} = E_{11}/8 \tag{1}$$

where E_{11} is Young's modulus of the composite when all the fibers are parallel to loading direction and continuous. Bishop[9] and Krenchel[10] have also been reported to derive similar results for very low transverse and shear moduli. Tsai and Pagano[11] have found the above isotropic properties through invariant properties of the composites. Christensen and Waals[12] derived the effective stiffness using an interesting model. However, they neglect the geometric details for macroscopic response. The common point of all the analyses is the assumption of homogeneous, anisotropic material instead of phasewise isotropic but non-homogeneous distribution of

materials. So far as macroscopic response of the system, namely the evaluation of elastic properties, are concerned the above assumption might not be a ~~real~~ gross assumption; but it may definitely lead to a non-conservative estimate in the sense that th detailed stress distribution estimated may completely be different from the actual one. For this reason only micromechanics study where exact phase distribution is used, has become very popular in the field of composites. Unfortunately this micromechanics study has been used only for regular array filamentary composites.[13-15] In this analysis the composite is treated as non-homogeneous but phasewise isotropic medium with random fiber distribution. So the local variations in the stress pattern due to different orientation and/or spacing of the fibers can be confidently predicted and consequently the mechanisms of crack initiation and propagation can be better understood. It is on this understanding a micromechanics study is undertaken to put more confidence on the values of in-plan elastic properties.

It is quite understandable that the future of composite materials, like many others, does depend on the success of their us as commercial materials. Most of the commercial composites are mad of chopped strand mat. Thus in the end product we have randomly oriented fibers in the plane of the composites. The method developed here gives the fabricator a means to assure his customer about the performance of the product since he knows both the microscopic and macroscopic response of the system now.

Although a few results are available to evaluate the isotropic properties of randomly oriented filamentary composites, this micromechanics approach adds a new dimension to them. Tsai and Pagano[11] have suggested to use these isotropic properties whenever 'designed anisotropy' cannot be exploited. They have concluded that this isotropic strength is the minimum one which can be achieved for any composite system. So this analysis can be equally used to estimate the minimum strengths of the composite with any combination of the matrix and the fiber(s).

The well-known Finite Element Methods are used for this micromechanics study. A brief description of the numerical technique is in the next Section.

2.2 Finite Element Methods:

Finite Element Methods are now extensively used in engineering practice. Quite a number of programs are available against payment.[16] Also there are at least three programs[17-19] freely available for solving a large number of structural problems. In addition, in every advanced centre of education and research, individual researchers are making their own programs. The method is based upon idealising a continuum structure with infinite degrees of freedom by a finite number of small sub-regions or divisions interconnected by a definite number of joints or points (the so-called Nodal Points), each with a predefined number of degrees of freedom. These regions or divisions are known as Finite Elements.

For detailed discussion of the finite element methods, the reader is referred to standard text books.[17,18] However, for the sake of completeness, the derivations of rectangular hybrid element and constant stress bar element are given below.

2.2.1 Pian's Hybrid Formulation

In this method the stress distribution in the element is assumed in terms of m (= 5 here) undetermined stress co-efficients $\{\beta\}$ as follows[23-25]

$$\{\sigma\} = [P]\{\beta\} \tag{2}$$

This assumed stress distribution must satisfy the equilibrium equations. Also compatible boundary displacements {u} are assumed.

$$\{u\} = [L]\{q\} \tag{3}$$

The surface force {S}, along the boundary of the element, can then be derived from (2) as

$$\{S\} = [R]\{\beta\} \tag{4}$$

From the stress-strain relation

$$\{\varepsilon\} = [N]\{\sigma\} \tag{5}$$

the following matrices can be introduced

$$[H] = \int_V [P]^T [N] [P] \, dV \tag{6}$$

$$[T] = \int_{A_2} [R]^T [L] \, dA \tag{7}$$

The total complimentary energy functional can then be expressed as

$$\Pi_C = \frac{1}{2}\{\beta\}^T [H]\{\beta\} - \{\beta\}^T [T]\{q\} \tag{8}$$

The condition of minimum complimentary energy is assured if[20]

$$\frac{\partial \Pi_C}{\partial \beta_i} = 0 \quad \text{i.e.} \quad \{\beta\} = [H]^{-1} [T] \{q\} \tag{9}$$

Substituting (9) in the expression of strain energy, it is found

$$U = \frac{1}{2} \{\beta\}^T [H] \{\beta\} = \frac{1}{2} \{q\}^T [T]^T [H]^{-1} [T] \{q\} \tag{10}$$

This means that the element stiffness matrix is given by

$$[k] = [T]^T [H]^{-1} [T] \tag{11}$$

The stress is calculated from (2), namely,

$$\{\sigma\} = [P] [H]^{-1} [T] \{q\} \tag{12}$$

In this analysis, for hybrid rectangular elements (fig. 1a), the equilibrating stress is assumed as:

$$\sigma_x = \beta_1 + \beta_2 y \; ; \qquad \sigma_y = \beta_3 + \beta_4 x \quad \text{and} \quad \sigma_{xy} = \beta_5 \tag{13}$$

The compatibility of displacements along element boundary is maintained by choosing the displacements as

$$\left.\begin{aligned} u &= \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy \\ v &= \alpha_5 + \alpha_6 x + \alpha_7 y + \alpha_8 xy \end{aligned}\right\} \tag{14}$$

Following the above mathematical steps, both the stiffness and stress matrices can be calculated. For the sake of convenience, [P], [R], [N], [T] and [L] are reproduced in the Appendix A. Since the final stiffness and stress matrices involve the matrix operations of these five basic matrices, they are done by the computer and hence not reproduced here.

2.2.2 Constant Stress Bar Element

In Displacement method, the 'exact' displacement in the structure is approximated by various degree polynomials. For Constant Stress Triangles (CST) only upto linear terms are taken. In terms of known model displacements, the assumed displacements are written as[17,18]

$$\{u\} = [C]\ \{q\} \tag{15}$$

From which the strain matrix can be calculated in the form

$$\{\varepsilon\} = [B]\ \{q\} \tag{16}$$

Using the stress-strain relation, it is found

$$\{\sigma\} = [E]\ [B]\ \{q\} \tag{17}$$

Assuming that no body forces act, the total potential energy is given by

$$\Pi_P = \frac{1}{2}\int_V \{q\}^T [B]^T [E] [B] \{q\}\ dV - \int_{A_1} \{q\}^T [C]^T \{\bar{T}\}\ dA \tag{18}$$

where $\bar{T}$ is the prescribed surface traction in the portion A_1 of the element.

If the principle of minimum potential energy is now used,[20] then it is found that

$$[k] = \int_V [B]^T [E] [B]\ dV \tag{19}$$

and

$$\{Q\} = \int_{A_1} [C]^T \{\bar{T}\}\ dA \tag{20}$$

The stress matrix can be calculated from (17).

For constant stress bar elements, it is assumed that the bar takes the axial loads alone. The displacement along the length of the bar is assumed as

$$u = q_1 + (q_2 - q_1) \cdot \frac{x}{L} \tag{21}$$

From (18), the matrix [C] is calculated as

$$[C] = \frac{1}{L} \begin{bmatrix} -1 & 1 \end{bmatrix} \tag{22}$$

For isotropic, homogeneous one-dimensional bar element [E] equals to E. So the bar element stiffness is calculated from (21) and (19) as

$$\frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{23}$$

where A and L are the cross-sectional area and the length of the bar respectively (fig. 1b). If the bar is inclined at an angle, the standard transformation[17,18] matrix is to be used to calculate the deflection pattern of the structure. However, for the problem at hand, no transformation is needed.

2.2.3 Assembly and Convergence

To solve for the deflection of the overall structure, all the element stiffness matrices are to be assembled as per the nodes and corresponding nodal degrees of freedom. For the hybrid rectangular elements, the degrees of freedom per node are 2. But for bar elements, the allowed degree of freedom in the problem at hand is one. So to assemble a bar and a rectangular element along the common nodes, a number of zeros are inserted

in the stiffness matrix of the bar element to account for the non-existing degree of freedom in the perpendicular direction of the bar. This is done automatically in the program. The scheme is shown in Appendix B with reference to fig. 1c.

It is noted that both types of elements are standard and well tested for convergence.[18,21,22,26] Actually for that reason only, they are picked up. So no special treatment is needed for the purpose. It is also hoped that on assembling these two types of elements in a single program the convergence is not affected; after all the total degrees of freedom per node is not changed. Due to the choice of the typical problem, the total number of elements in the overall structure becomes fixed. The number of elements cannot be reduced to keep the structure unchanged. On the other hand, the number of elements are too large to be increased further without crossing the available computer memory. So no direct proof of the assembled convergence for the specific problem could be done. The only comforting point is that the chosen elements are small enough and in addition, an indirect proof of convergence is given in Chapter V. Also, a number of standard problems with known solutions have been tested with the developed program. The results give equally good, if not better, results for these problems. The details of the program and its performance are given in the next chapter.

CHAPTER III

THE PROGRAM AND THE PROBLEM

3.1 The Master Program:

A general purpose program has been developed for structural analysis under static loading with a switching character of using either CST or hybrid or both types of elements. For solving the large number of simultaneous algebraic equations, Wilson's BANSOL[19] is used unchanged. Rectangular displacement elements are formed by static condensation from four equivalent constant stress triangles (CST). Rectangular hybrid elements[23] are used with first five terms using linear normal stress variation and constant shear stress. However, extension of the hybrid elements for higher order polynomials is possible without much trouble.

The program is then used for solving the problem of pure bending of a beam given in Ref. 18, pp. 165-167 (fig. 2). The results are obtained for both displacement and hybrid elements and compared with the exact solution in Tables 1 and 2. ~~As~~ It is, now, obvious that the hybrid formulation gives quite exciting result for this particular problem. Almost exact values of displacements and stresses are reached even with such a limited number of elements. The time taken is also slightly more than that required by displacement method for same number of nodes.

With these encouraging results, the basic problem of evaluating elastic properties by finite element method is taken up.

As another test check, Young's Modulus E and Poisson's ratio ν ~~is~~ are evaluated by displacement and hybrid method for an isotropic, homogeneous material. Uniform tension is given at the ends of a typical test specimen of 2" length and a square cross-section of 1" x 1". The response of the system is calculated by finite element methods. Actually it is the computer simulation of the tension test Changing the material property and aspect ratio (length by width), if necessary, the elastic properties can, thus, be calculated for any material, homogeneous or non-homogeneous, isotropic or ortho-tropic or anisotropic. The only requirement for evaluating material properties by this method is a general purpose program which has been successfully developed. Obviously, a good computer is needed for getting the results.

In the computer simulation of the tension test, the uniform tension is replaced by a number of concentrated loads at the nodal points, the value of which is calculated by consistent loadings. The thickness of individual materials and the specimen is always taken as unity. In the program, provision is made to use upto twelve different materials. The advantages of automatic node and element generations have also been used. The total number of nodes and/or elements does depend on computer memory. All the calculations of the present analyses have been done with the IBM computers of I.I.T., Kanpur (7044) and V.S.S.C., Trivandrum (360). The maximum number of nodes, each with two degrees of freedom, is kept as 400. In this program, arithmetic average has been used to calculate the average stress and average strain. After all,

Young's modulus is the manifestation of the macroscopic response of a system under static loading.

For the isotropic, homogeneous material, 25 nodes and 16 elements have been used to estimate the elastic properties. From the deformation pattern, it has become obvious that larger the nodal number, the better the average. So finer mesh always gives a better result until it converges with the exact one. This is particularly true for evaluating Poisson's ratio. The numerical results of the elastic properties are given in table 3. The interesting feature to note here is as follows. So far as elastic properties, in general, are concerned, both the displacement and hybrid methods give almost the same result. But, it is already pointed out that the detailed stress and deformation pattern are quite different for these two formulations. This implies that for elastic properties, the displacement method is also equally good. However, for micromechanics study it is always preferable to go for hybrid method.

From table 3, it is apparent that the values of Young's modulus obtained from either formulation is very close with the exact one. However, Poisson's ratio varies slightly from the exact value. A finer mesh, giving a better average, will obviously lead to the exact value. It may be noted that this property is rather difficult to evaluate even experimentally.[7] Fortunately, the effect of Poisson's ratio is not very much sensitive for structural analysis!

3.2 Continuously Reinforced Composites:

The method of evaluating elastic properties by this computer simulation technique is then extended for continuously reinforced composites. There are a large number of formulae available for the prediction of longitudinal and transverse Young's moduli .[5,6,27-29] Obviously all these results have come up because exact values for these parameters cannot be predicted due to associated problems like complex microstructure, interface characteristics, voids, fiber shape, size, distribution, alignment and continuity. These difficulties have made the standardisation of experimental techniques also very difficult. Consequently, the experimental results show a certain amount of deviation as shown in fig. 3. So to measure/predict the elastic constants, both the theoreticians and experimentalists are forced to accept a number of assumptions. The assumptions made in this analysis include perfect bonding and absence of voids. The detailed assumptions and their implications are treated later.

The details of the fiber orientations and loading systems for longitudinally and transversely reinforced composites are shown in figs. 4a and 5a. A very crude model with nine nodes only is used to evaluate the longitudinal and transverse elastic moduli of continuously reinforced composites. Even with this crude model, the agreement with other results is quite reasonable. The finite element mesh and deformation patterns for these two specimens are shown in figs. 4b and 5b. The results are summerised in table 4.

It is quite interesting to note again that Young's modulus obtained in two finite element methods are the same.

It is generally accepted that Paul's prediction for longitudinal Young's modulus is perhaps the simplest and most widely applied result in the field of composites. The slightly higher value obtained in finite element method is due to the crude mesh. It is believed that this will converge to Paul's result if finer mesh is chosen. To this context, the result of Ref. 28 is rather doubtful. On the other hand, the transverse Young's modulus obtained by the finite element methods is very much acceptable. Considering the variations in the experimental results,[5] the result can very well be deemed ~~demanded~~ as the correct one.

With all these data generated and compared, the program and its applicability in the field of composites have been fully assured. So the actual problem outlined in the introduction is being stated and systematically analysed now.

3.3 The Problem and Solution Technique:

In chopped strand mat short, discontinuous reinforcements are randomly distributed in the plane of the composite (fig. 5a). Also composites with natural fibers or low-cost-high-sales-volume commercial composites, in general, often have randomly oriented in-plane fibers. The much publicised low-density high performance filamentary composites mostly owe their desired strength in the exactness of the knowledge of load distribution. On the other hand, if the loading history is not fully known, as is the case in many

real life applications, the anisotropic characteristic of these materials cannot be exploited. Actually in such cases, it is always preferable to start with isotropic properties of randomly oriented fibers for reliable, optimal design. This type of isotropicity has been recommended to introduce through the thickness for launch vehicle applications.[30] Similar isotropicity can be achieved with four suitably oriented laminae (fig. 6b). Tsai and Pagano[11] have suggested The ~~to~~ use of these isotropic properties as a measure of minimum stiffness capability which can be confidently used whenever, the advantage of designed anisotropy cannot be exploited.

To facilitate both the microscopic and macroscopic analyses of such an important class of composites, finite element method is chosen as the solution technique. Young's modulus and Poisson's ratio are calculated by simulating pure tension test of a randomly oriented filamentary composites in the computer and then taking the arithmetic average of the test specimen. Shear modulus is calculate using the standard formula for isotropic material

$$\bar{G} = \bar{E}/2(1 + \bar{\nu}) \qquad (24)$$

The bar indicates ~~implies~~ the average value.

To simulate a tensile test specimen of randomly oriented filamentary composites, the fibers are to be randomly distributed in the plane of a typical test specimen of matrix material. To orient the fibers randomly in the plane of the composite, random numbers are generated from the standard library program of the

computer. Equal probability for any orientation out of n possible orientations is chosen. So, the total span of the random numbers, greater than zero ~~to~~ and less than unity, is divided equally in groups, 1, 2, ... n. As a particular number is generated, the correspon-ding group for the number is identified. Since every group corresponds to a particular orientation, the orientations of the fibers are systematically identified from the generated random numbers and accordingly placed in the matrix test specimen. In this particular analysis, only longitudinal and transverse orientations have been chosen. Also it is assumed in the analysis that the fibers act as bars. So in analysing the problem by finite element method, the fiber volume fraction is to be introduced through the cross-section and/or length of the short, discontinuous fibers.

After careful consideration of the tensile test specimens used for conventional, plastic and composite materials, the dimension of the simulated test specimen is chosen as 8" x 2" x 1". The thicknesses of fiber, matrix and test specimen are all unity. In addition, it is decided to keep the length of all the fibers equal and as unity. But to justify the assumption of bar element for the fibers, the fiber aspect ratio (length by width) should be much larger than unity. This implies that all the fibers are plates (fig. 7a). Now, to solve the problem of fiber volume fraction, the test specimen is divided into sixteen blocks (fig. 7b). Each of these sixteen blocks is filled with fibers of identical orientation. The number of fibers in a block depends on the fiber

volume fraction and chosen fiber width. As for example, for a chosen width of 0.1", corresponding to fiber aspect ratio 10, the number of fibers in each block will be 3 for a fiber volume fraction 0.3. Thus a methodology is established for simulating randomly oriented filamentary composites with the help of a computer.

Following the methodology and using the simulated tension test both the microscopic and macroscopic responses of randomly oriented filamentary composites can be studied. It is also decided to study the effects of a few important parameters on the macroscopic response of the material, measured through Young's modulus. The parameters considered include randomness, biasness, volume fraction and residual stress.

3.4 Assumptions:

Identifying the problem, the following assumptions are made for the analysis:

(1) Both the matrix and the fibers are isotropic, homogeneous materials in bulk and in-situ.

(2) All the plate-like fibers are geometrically identical.

(3) The composite is free from voids.

(4) All the fibers are perfectly bonded initially and remain so during loading.

(5) The composite is in a state of plane stress and obey Hooke's law.

In addition, to solve the problem by finite element method, the following assumptions are also added.

(6) Effects of stress concentration, at the ends of the fibers, are not considered explicitly.

(7) The fibers take only axial loading and the axial load developed is constant all through the length.

Since the composite is in a state of plane stress, the plate-like fibers do not disturb the in-plane stress distribution. So, even with curious geometries, the fibers are representative. Actually, analysis with similar fibers has been done successfully.[13] The other assumptions regarding the composite have proved to be realistic many a times.

Although the effect of stress concentration has not been considered explicitly, it is believed that the elements near the fiber tips have been chosen small enough to arrest the steep stress gradient effectively. The fibers donot have any stiffness in the transverse direction in bulk. So the fibers in-situ too can be treated as bar, particularly if the fiber aspect ratio is kept high. For discontinuous fibers, the axial stress is not actually constant in the fibers. It is constant beyond the critical length only. The constancy of axial stress implies an effectively longer fiber. However, this slight increment in effective fiber volume fraction is rather compensatory since the treatment of the fibers as lines in the finite element mesh effectively increases the volume of the matrix.

The results of this analysis are, now, being discussed in the next two chapters. Before studying the effects of different parameters on the performance of randomly oriented filamentary composites, the next chapter is completely devoted to the detailed study of a typical case.

CHAPTER IV

MICROMECHANICS STUDY OF RANDOMLY ORIENTED FILAMENTARY COMPOSITES

4.1 Introduction:

If high strength short discontinuous fibers are randomly distributed in a low strength molten matrix and the whole system is cured at a specific temperature and pressure, the resulting randomly oriented filamentary composite is an isotropic material. This type of isotropic material is made, in particular, with natural fibers like coir, asbestos and sishal. In the commercial applications of composites with chopped strand mat (CSM), the end products are similar isotropic materials. Thus this type of composites constitutes an important class of materials which need to be carefully analysed. Also it may be noted that whenever the designed anisotropy of the composites cannot be successfully utilised, the best solution is to use this type of isotropic composites.

The material properties of similar composites were first evaluated by Cox.[8] But no account was made for the matrix. So the analysis applied to paper are not reliable for modern composites particularly when the matrix carries a part of the total load. Krenchel[10] is reported to have derived similar results for very low transverse and shear modului. The isotropic properties were derived by Tsai and Pagano[11] also using invariant properties. In

a rather recent report Christensen and Waals[12] have derived similar result based on a model of equivalent volume fraction for randomly oriented fibers.

The one important common feature in all the above analyses is the derivation of elastic properties of the composite without considering the local inhomogeneity of randomly oriented short, discontinuous fibers. So far as the macroscopic response of the composite system is concerned, the above theories predict it within reasonable limits. But to the authors best knowledge, no theory exists which ~~to~~ considers the local inhomogenieties. It is noted that the most widely used assumption of anisotropicity homogeneity of the composites cannot predict the local stress variations since the actual system is phasewise isotropic but distribution of the phases is non-homogeneous. But the local stress within a composite may vary widely depending upon the local inhomogeneity and it is the local stress variations which control the initiation and propagation of cracks causing the ultimate failure of the structure.

4.2 Analysis and Assumptions:

To accomplish the detailed study of the stress distribution, a micromechanics study is essential. Finite element method is used for the purpose. The matrix elements are simulated through five term Pian hybrid[23,24] rectangular elements. The fibers, assumed to take only axial loads, are taken as constant stress bar elements.[17,18] A general purpose program has been developed for solving structural problems with statical loadings by the use of hybrid or displacement

or both types of elements.

For micromechanics study of two-phase randomly oriented short, discontinuous filamentary composites, a typical tension test specimen (aspect ratio = length/width = 4), is subjected to pure tension and the corresponding response of the specimen is studied at different cross-sections. To simulate the random fibers with specific volume fraction, the test specimen is subdivided into 16 equal 1" x 1" cells (fig. 7). In each cell the fibers are unidirectional and they represent the exact volume fraction of the total system. To match with the memory capacity and also to maintain the fiber volume fraction of 0.3, three fibers were chosen in each cell. For all the fibers, the geometry is identical. To simulate randomness, the random numbers are generated from the computer with uniform probability. Equally it can be extended for any other statistical distribution. For the sake of simplicity only two orientations of the fibers (longitudinal and transverse) are chosen. If memory permits, four orientations (0°, 45°, 90° and 135°) are sufficient to exactly simulate the randomness. However, the results seem to be equally good with two orientations only. Using again equal probability, the orientations of fibers in all the cells are decided from the generated random numbers. The final simulated structure of the 'isotropic system' is shown in fig. 8a. Since plane stress condition is assumed for the test specimen, the geometry of the fiber, chosen as unit length, thin plates, does not really matter. The thickness of the test specimen is chosen as unity.

The isotropicity and homogeneity of both the matrix and fiber are assumed for bulk and in-situ conditions. Also the fibers are assumed to be perfectly bonded and there is no void in the composite. In addition, the fibers are supposed to take axial loads alone and the axial stress developed is constant all through the length. The effect of stress concentration at the end of the fibers is not treated explicitly. However, the finite elements near the fiber tips are chosen small enough to arrest the steep stress gradient effectively. The whole analysis is limited in the elastic region.

The tensile test specimen is subjected to uniform tension. The uniformly distributed tensile stress is lumped at the nine modes of the final finite element mesh of the specimen by the use of consistent loading. The finite element mesh, the co-ordinate system and the consistent loading is shown in fig. 8b.

4.3 Results:

The response of the test specimen under uniform tension of 4000 psi is calculated by the general purpose program developed. The material properties are given below

Matrix (Epoxy Resin)

Young's Modulus = E_m = 0.4×10^6 psi and

Poisson's Ratio = ν_m = 0.35

Filaments (Glass Fiber)

Young's Modulus = E_f = 11.8×10^6 psi and

Poisson's Ratio = ν_f = 0.197

Cross-sectional area of each fiber = 0.1 sq. inch.

It may be noted that the fiber depth being unity in each case, the fiber width is 0.1 inch for this case.

$\therefore$ Volume fractions of the filaments = V_f = 0.3.

4.3.1 Deflection Pattern

The vertical displacement of the top and bottom nodes and the horizontal displacement of the side nodes are shown in fig. 9. As expected, the longitudinal deformation of the test specimen is not uniform. The deformation is less along all the nodes where fibers are placed longitudinally. In other nodes, matrix takes most of the load since the fibers are transversely placed. The matrix being a weaker material, the deflection is more along these nodes.

The transverse deformation has a distinct bending pattern, eventhough the loading is pure tensile. To pinpoint the bending effect, the horizontal deformation of the side nodes is separated (fig. 9) from the deformed test specimen shown in fig. 8c. Obviously the coupling between pure stretching and bending is due to the local inhomogeneities. The inhomogeneity gives rise to the shift of the centroid from the plane of action of the resultant force. The shift of the centroid also varies from section to section as shown in fig. 10. Depending upon the shift of the centroid, an in-plane bending moment is produced which pushes the nodes horizontally. The direction depends on the sign of the bending moment which obviously depends on the nature of local inhomogeneity. Accordingly the deflection pattern may take any curious shape (fig. 8c)

depending on the random distribution of the filaments. Also it should be noted that the Poisson's effect plays an equal role to determine the final deformed structure of the 'test specimen'. The transverse fibers always pick up the transverse load, irrespective of its source and thereby limits the absolute magnitude of horizontal deformation. It may be of interest to verify the in-plane bending effect for randomly oriented filamentary composites, both qualitatively and quantitatively.

4.3.2 Strains

Fig. 8c shows the deformed structure of the test specimen simulated by the computer for a randomly oriented filamentary composite. The deformation is arithmatically averaged out (as shown by dotted line) to calculate the macroscopic response of the specimen. The deformation is magnified hundred times in the figure. For 8" long test specimen, the average longitudinal strain is found to be 0.426×10^{-2} for a uniform tension of 4000 psi. The detailed variation of longitudinal and transverse strain in the specimen is shown in fig. 11. To calculate longitudinal strain, different gauge lengths are chosen, each measuring from the top plane ($y = 0$) of the specimen. The longitudinal strain is pure stretching. The average longitudinal strain calculated from the graph, is expectedly almost identical with overall strain measured from average deformations of the whole test specimen. The variation is only about 6 per cent. On the other hand, the transverse strain is influenced by in-plane bending. So simple arithmetic average might be

misleading to calculate the transverse strain.

To take care of the bending effect in calculating the transverse strain, rather the Poisson's effect, ideally an integration technique is to be used from point to point deflection. Instead the graphical method is used here. The transverse strain is calculated at different cross-sections corresponding to different values of -y and plotted. The average strain is then calculated graphically. At transverse sections, corresponding to -y = 1", 2", 3" etc., the transverse strain is calculated here and plotted in fig. 11. From this graph average transverse strain is calculated. It may be noted that the two strains vary by an order of magnitude almost. From these strains, Poisson's ratio is calculated as

$$\bar{\nu} = \frac{\bar{\varepsilon}_x}{\bar{\varepsilon}_y} = \frac{0.060 \times 10^{-2}}{0.426 \times 10^{-2}} = 0.1462 \simeq 0.15 \qquad (25)$$

There is a definite variation of either strain at different planes or gauge lengths, but nowhere the variation is alarmingly sharp. This assures that the variation is due to local inhomogeneity and there is no strain concentrations.

4.3.3 Stresses

In continuous filamentary composites, the stronger fibers always take most of the load. The role played by the weaker matrix is to bind the fibers together and protect them from environmental and frictional degradation of strength. The matrix also assures a transverse stiffness which the fibers alone cannot offer. For short discontinuous fibers, the normal stresses in the fibers are

developed through the shear stress of the matrix. Thus shear stress plays an important role for short discontinuous filamentary composites. For randomly oriented filamentary composites, the fibers are effective, that is they carry most of the load, only if they are placed along the direction of normal stress. If they are placed transversely with respect to the normal stress, they donot contribute anything to ~~share~~ The sharing of the normal stress. In such cases matrix need to carry the whole of the normal stress developed.

Fig. 12 shows a typical distribution of the normal stress, σ_y, developed in the direction of loading. The stress developed is normalised with respect to the average stress applied. The cross-section is DD at a distance of 2.625". The different cross-sections mentioned herein are shown in fig. 9. It is clear from the figure that the stress in the matrix is hardly 10 per cent of the applied stress wherever the fibers are there to pick up the load. The stress in the fibers goes as high as 4 times the applied load. On the other hand, the fibers donot carry any load wherever the fibers are transversely placed (right part of fig. 12). Consequently the applied average stress is quickly developed in the matrix. The stress concentration effect is also quite clear from the figure. The maximum stress concentration factor is found to be 4.5. It is noted that this value is approximately double than that ~~for~~ of regularly spaced fibers.[14]

The in-plane bending moment generated due to local inhomogeneity causes tension and compression along the length of the specimen. Consequently, σ_y also varies along a longitudinal

plane corresponding to a fixed value of x, even if a constant tensile stress is applied. Fig. 13 shows the variation of σ_y along two longitudinal planes. Thus it is noted that the variation of the stress distribution is very much controlled by the bending stress, rather the nature of local inhomogeneity.

The distribution of σ_x is rather curious. As mentioned already, the distribution of σ_x is controlled by Poisson's effect and the in-plane bending effect caused by local inhomogeneity. But the local inhomogeneity varies from plane to plane for a randomly oriented filamentary composite. Also the quantitative value of σ_x depends, in addition, on the overall distribution of other fibers. Figs. 14-16 show the normalised stresses, σ_x and σ_{xy}, at different cross-sections. The stresses are normalised with respect to applied stresses and plotted as per cent values. As in case of σ_y, σ_x is also mostly taken by the transversely oriented fibers. This means that the transverse fibers, ineffective ~~to take~~ in taking the applied load, now takes σ_x. Thus for randomly oriented fibers, the composite is never adversely weak in any direction. At least a particular percentage of fibers is always there to pick up the load coming from any direction; thus making the composite strong in every direction. Hence the randomly oriented filamentary composite can safely be used whenever the designed anisotropy cannot be exploited. On the other hand, however, since the matrix will have to carry a part of the load, the fullest exploitation of the fibers cannot be employed. So the ultimate choice of the composite depends on both the magnitude and direction of the applied forces.

It is interesting to note the antisymmetry of σ_x distribution in the planes of D-D and G-G (fig. 16), EE and FF (fig. 15). The same antisymmetry is also present for the shear stresses in those planes. These antisymmetries reflect the opposite directions of the in-plane bending moments originated due to the shift of the centroid of the inhomogeneous plane in opposite directions. This implies that from the nature of the distribution of σ_x and σ_{xy}, definite conclusions about the nature of the local inhomogeneity can be drawn. Otherwise, if the nature of the local inhomogeneity is fully known, the possible variation of σ_x and σ_{xy} can be anticipated. From the analysis, it is found that maximum tensile stress in the transverse direction has developed in the matrix and equals to 40 per cent of the applied stress. On the other hand, the maximum compressive stress has developed in the fiber and equals to 145 per cent of the applied stress. It may be recalled that compressive stress is rather beneficial for better bonding of the fibers in the composite.

It is assumed in the analysis that fibers take only axial load. But the matrix elements are allowed to develop the shear stress which is assumed to be constant. Consequently, an estimate of the maximum shear stress developed along the interface for the load transfer mechanism can be made which is valid only if the bond between the fiber and the matrix is perfect. The maximum shear stress developed in the composite is found to vary from -30 to +40 per cent of the applied stress. Since the shear strength of any material is much lower, the shear stress developed in the

composite can cause concern. Thus, for randomly oriented filamentary composite, the shear strength may become a controlling factor. Once a crack is developed due to shear failure, the crack may be propagated across the composite cross-section by the transverse normal stress, σ_x. This only implies that care must be taken, by complete stress analysis, in designing randomly oriented filamentary composite. A complete analysis alone can safeguard the applicability of this group of composites. The detailed stress analysis, thus, forms an important part in the design of this group of composites, in particular.

4.3.4 Macroscopic Response

The macroscopic response of the composite is finally evaluated by arithmetical averaging process. The actual and averaged deformation pattern of the composite is shown in fig. 8c. The applied stress is 4000 psi. So the average Young's modulus can be calculated. The average transverse strain is calculated from the graph. The calculated values of different parameters are given below:

Average longitudinal deformation = 0.03410893" (from analysis)

$\therefore$ Average longitudinal strain = 0.426362×10^{-2} (calculated)

Average applied stress = +4000 psi (applied)

Average Young's modulus = 0.94×10^{6} psi (calculated)

Average transverse strain = 0.060×10^{-2} (from graph)

Average Poisson's ratio = 0.146 (calculated)

Average shear modulus is calculated as follows:

$$\bar{G} = \frac{\bar{E}}{2(1 + \bar{\nu})} = \frac{0.94 \times 10^6}{2(1 + 0.146)} = 0.41 \times 10^6 \text{ psi.}$$

Table 5 shows the predicted values for Glass-Epoxy system and compares with other available results.

Unfortunately, no experimental results are available for comparison. Christensen and Waals[12] have compared their results with experimental results (Fig. 17) for low fiber volume fractions. The material properties used are close to the one used in this particular analysis. But results are not available for comparison for $V_f = 0.3$. However, from the trend in the figure shown, it seems that the difference between the predicted and the experimental result is quite large for higher V_f. Also a comparison of the experimental results with the prediction of Tsai and Pagano is available as shown in fig. 18.[31] The comparison is made for random short length boron filament reinforcement in epoxy matrix. For $V_f = 0.4$ and low L/D ratio (= 10 here), the predicted values are definitely higher than the experimental one. Under the perspective, the results obtained by this method are believed to be equally, if not more, reliable than the existing ones. However, the real advantage of the method lies in detailed micromechanistic study.

4.4 Conclusions:

From the ~~previous~~ present analysis, the following conclusions can be summarised

(1) A methodology has been established to simulate the randomly oriented filamentary composites with the help of the computer and finite element method has been successfully used for the micromechanics study of a computer simulated test specimen. Pian hybrid method demands more extensive study for application in the field of composites.

(2) The simulated test specimen has shown a distinct effect of in-plane bending due to local inhomogeneity. The effect might be smoothened out if the number of blocks in the transverse plane could be increased substantially. However, it might be equally interesting to find the possible bending effect, its nature and magnitude, through careful experimental measurement.

(3) The effect of stress concentration is predominant for randomly oriented fibers. For the glass epoxy system it goes as high as 4.5. Similarly the effect of shear is also very high for random distribution. The shear stress may go as high as 40 per cent of the applied stress. So a detailed micromechanics study is particularly helpful ~~to~~ in the use of randomly oriented filamentary composites with confidence.

CHAPTER V

PARAMETRIC STUDY OF RANDOMLY ORIENTED FILAMENTARY COMPOSITES

5.1 Introduction:

The future of composite materials, like many others, does not depend on some specific, scientific applications. Rather it does depend on the possibility of extensive, commercial applications of the material. Again for successful commercial application, the confidence about the material performance must be gained through systematic, scientific analysis. If the fibers are randomly oriented in a matrix, the end product will be isotropic and homogeneous. But, obviously, the strength of the composite will not be very high in a particular direction. On the contrary it will be equally strong in any direction. So, this isotropy is useful whenever the designed anisotropy cannot be used. Also, for a safer, optimised study these isotropic properties can be used as a starting point. In short, the randomly oriented filamentary composites have an important role to play in both commercial utilisation and scientific investigation.

But unfortunately, very little has been done for this group of materials. Cox,[8] Krenchel,[10] Tsai and Pagano,[11] Christensen and Waals[12] have derived individually the different in-plane elastic properties of the composite. To assume phasewise isotropic but distributionwise non-homogeneous composite material as

anisotropic, homogeneous material is the common feature in all the derivations. Although the methods lead to rather reliable results, the detailed stress distribution in the composite cannot be achieved by any of the methods. A microscopic approach has already been discussed for microscopic and macroscopic analyses of randomly oriented filamentary composites. In addition, to the author's best knowledge, no systematic analysis is available to assess the effects of a number of important parameters on the performance of this group of composites. Here an attempt is made, with the help micromechanics study, to study the effects of randomness, biasness, fiber volume fraction and residual stress on the in-plane elastic properties of the composite. Tsai and Pagano[11], Christensen and Waals[12] have studied the effects of volume fraction in their reports.

5.2 Assumptions and Analysis:

The tensile test specimens of randomly oriented filamentary composites is simulated by the computer and subjected to pure tension within the elastic range. The response of the specimen is obtained using finite element method. The matrix elements, capable of taking linearly varying normal stress and constant shear stress, are chosen from Pian's five term hybrid element. The fibers are assumed as constant stress bar elements which can take only axial load. Both the fiber and matrix are assumed to be isotropic, homogeneous material and are perfectly bonded. No void is assumed to exist.

It is already mentioned that the study is limited on the effects of randomness, biasness volume fraction and residual stress on the in-plane elastic properties of randomly oriented filamentary composites. This group of composites are fabricated by randomly distributing short, discontinuous fibers in a layer of molten matrix and subsequently curing the whole system at a specific temperature and pressure. Since no control is made in distributing the fibers in the matrix, the fiber placement definitely varies from specimen to specimen. Similarly, in the simulated test specimen, the fiber orientation also may vary depending on the random number generated and/or their placement, longitudinally or transversely. But, true randomness always converges to an isotropic system. So, for a particular set of matrix and fiber, the composite material should always be the same isotropic and homogeneous material provided the above mentioned assumptions are adhered to.

In most of the cases the isotropic behaviour of composites with random reinforcement is used whenever the loading direction is either unknown or arbitrary. On the contrary, if the loading direction is not arbitrary and also fully known, the designed anisotropy of the composites is fully exploited. It is of interest to investigate the intermediate phases between the isotropic and designed anisotropic phases. In terms of statistics, for the case of isotropic composites, the probability of fiber orientation is equal in any direction. For designed anisotropy, the probability of particular fiber orientation(s) is (are) specifically higher

than others. In this study, only two fiber orientations are used. The random numbers generated, which lie between zero and unity, are accordingly divided into equal or biased groups to represent the fiber orientations. For isotropic composites the probability of fibers in the direction of loading is used as 50 per cent. As the biasness increases, the probability increases. Here the probabilities used are 60, 70, 80 and 90 per cent. In addition, fibers oriented in the direction of or perpendicular to the direction of loading are also analysed to represent the extreme cases of 100 and 0 per cent probability respectively.

The effect of increasing fiber volume fraction can be simulated by increasing either the number or the cross-section of the fibers. But unfortunately, due to limitation of computer memory, the number of fibers could not be increased to increase fibe volume fraction. So the cross-sectional area of individual fibers has been increased and subsequently analysed to study the effect of increased volume fraction. It is noted that increase of volume fraction by this method is highly artificial, particularly when the fibers are assumed as bar elements capable of taking axial loads alone. However, it is hoped that this shows the trend of the effect of increased volume fraction.

Due to differential thermal expansions of the matrix and the fiber, a residual thermal stress is always developed. It is of interest to study this effect on the performance of the composite. In this analysis, the fibers are assumed to act as bar elements which can take the axial stress alone. So the residual

stress is incorporated as the initial stress in the fibers.

The same glass-epoxy composite test specimen (8" x 2" x 1") is chosen for the parametric study. The detailed response of the 'test specimens' are now analysed systematically.

5.3 Parametric Effects:

5.3.1 Randomness

True randomness always converges to a specific isotropic system of composites. The simulation technique should also equally converge to the same composite for identical matrix and reinforcing materials provided all the assumptions are adhered to in the process of fabrication of the composite and in addition the random numbers implying random fibers are sufficiently large. The second condition is somewhat important in the contradicting contexts of convergency and computer memory.

To identify random fiber orientations, a set of random numbers are generated. Now filling the sixteen blocks of a test specimen of the matrix transversely and longitudinally with fiber of volume fraction 0.3, two randomly oriented filamentary composite test specimens are made. The test specimens are then put under a uniform tension and averaging the response of the test specimens, the Young's modulus is calculated.

Fig. 8a shows the test specimen simulated by the computer when the random fibers are placed transversely. Figs. 8b and 8c shows the finite element idealisation and the response of the test specimen under uniform tension. The Young's modulus is calculated

as 9.4 x 10^5 for this specimen. The second test specimen and its response to a uniform tension of 2000 psi are shown in fig. 19 when the random fibers are placed longitudinally. The calculated Young's modulus is 9.6 x 10^5 psi. In this particular case, all the elements have been taken as displacement rather CST elements. It may be recalled that in either the displacement or the hybrid model, the calculated value of Young's modulus may differ slightly, if at all.

Noting the wide variations in the two test specimens, it may safely be concluded that the sixteen blocks are sufficient for showing the convergence of the material properties, at least with this set of random numbers. It is noted in this context that no convergence of the material response has been shown in the application of the finite element method. Actually no choice is left for showing the formal convergence. The computer memory is the only reason for which finer mesh could not be taken. On the other hand, the structure of the test specimen is so complicated that coarser elements could also not be taken. However, the comforting point has been that the elements are small enough. Now, with this randomness, the convergence is proved indirectly. The wide variation in structures rules out the possibility that the convergence of the test specimens to identical isotropic material is coincident. Rather it proves that, as expected, the randomly oriented filamentary composites have identical isotropic properties irrespective of its origin provided the above assumptions are adhered to. So, even if, the structural details may vary from

specimen to specimen, the end products are the same for random fibers.

5.3.2 Biasness

It is noted that although equal probability is assumed in aligning the fibers of the computer simulated test specimen, the number of fibers in the direction of loading is more than that in the transverse direction. It is also noted that the respective proportions are $\frac{5}{8}$ and $\frac{3}{8}$. These figures are used in calculating the isotropic properties of randomly oriented filamentary composites as derived by Tsai and Pagano.[11] Physically also it is quite understandable that effectively the load carrying capacity of randomly oriented fibers will be definitely more than $\frac{1}{2}$ since all the fibers except the ones in transverse direction will carry a part of the load, the share being dependent on fiber orientation.

Now, instead of dividing the random numbers generated equally and then using then to find the orientation of the fibers in all the blocks (sixteen in total) sequentially, they can be divided into unequal sets and use similarly to find the new orientations of the fibers. The physical interpretation is that more fibers are being oriented in a particular direction, say the direction of loading. This method of biasing the fibers in the known direction of loading and thereby increasing the stiffness of the composite in that direction is termed as designed anisotropy. As the biasness is increased, the stiffness is also increased. It is of interest to see how the two are related. To the author's

best knowledge no such relation exists.

The basic idea is to find a possible correlation. So such a set of random number is chosen, which on equal division gives almost equal numbers, 9 and 7, corresponding to the two directions. Also in addition, deliberately more number of fibers are oriented in transverse direction rendering them ineffective. It is then termed as 50 per cent biasness. Since the random numbers generated are between zero and one, 50 per cent biasness means that the numbers less than 0.5 and numbers equal to or greater than 0.5 represent the two perpendicular directions of fiber orientations. In a similar way biasness of any other percentage can be defined. It is again noted that this artificial method is deliberately chosen just to simulate the biased composites and consequently try to find a correlation between biasness and stiffness of randomly oriented filamentary composites.

Fig. 20a shows the simulated composite with 50 per cent biasness. Fig. 20b shows the deformation pattern of the composite test specimen when subjected to a uniform tension. Here all the nodes of loading are made free. From the deformation pattern, the Young's modulus is calculated. Figs. 21 to 24 show the test specimens with varying biasness of 60, 70, 80 and 90 per cent. All the test specimens are subjected to a uniform tension. Here all the nodes of loading are horizontally restricted. The corresponding deformation patterns are also shown. The deformations are magnified 100 times. The composite with all the fibers placed transversly to the loading direction and the

consequent deformation pattern under uniform tension is also shown in fig. 5. This may be treated as 0 per cent biasness. It is noted that the fiber volume fraction is always maintained as 30 per cent. The Young's modulus for all the biasness are calculated. For 100 per cent biasness (when all the fibers are in the direction of loading), the modulus is computed from the rule of mixture. Computing the relative modulii for different biasness with respect to the modulus of 0 per cent biasness, a plot is made between the relative Young's modulus and biasness (fig. 25). The interesting plot shows a regular trend of increasing modulus with increasing biasness. Also the rate of increment of the modulus increases as the biasness increases. The relative increase is almost nil for the first half of increase in biasness. On the other hand the increment is almost exponential for increasing biasness beyond 50 per cent. However, the curve is rather qualitative.

Another interesting point should be noted in the study of the effect of biasness. It is already mentioned that the local inhomogeneity gives rise to in-plane bending moment in the test specimen. But increase in biasness, in general, means a reduction in local inhomogeneity. As the local inhomogeneity decreases the bending effect also reduces (figs. 20-24) and as such, there is no bending effect when the fibers are continuous and placed longitudinally or transversely (figs. 4 and 5). Thus increase in biasness decreases the in-plane bending effect in addition to the increase in Young's modulus in the direction of the biasness.

5.3.3 Volume Fraction

It is well-known that volume fraction plays an important role in composites, in general. If the fibers are continuous and aligned in the direction of loading, the Young's modulus increases linearly with increasing volume fraction (rule of mixture). But the relation is not so simple for transversely placed filamentary composites. The effect of increasing volume fraction on Young's modulus has been calculated from the macroscopic response of randomly oriented filamentary composites by Tsai and Pagano,[11] Christensen and Waals[12]. Their results have been reproduced in figs. 17 and 18.[31] It is noted that the slope of the curve of Young's modulus - vs - volume fraction is about half for the random fibers than that for continuous composites with fibers in the direction loading.

The effect is investigated in this analysis also. But it is already mentioned that for the problem of computer memory the increase in fiber volume fraction is artificially simulated by increasing the cross-sectional area of all the fibers. Obviously this artificial technique cannot be used for very high volume fractions. In those cases, the fibers should also be treated as rectangular element instead of bar elements.

As the volume fraction is increased by increased cross-sectional area alone, the deformation pattern of the test specimens does not change. Only the average increase in length decrease with increase in volume fraction. As a result, figs. 8a

and 8c are representative of the test specimens and their deformation pattern.

Fig. 26 shows the effect of increasing volume fraction on Young's modulus of randomly oriented filamentary composites. The rate of increase in the modulus is rather low. It may be noted that corresponding increase for longitudinally continuous filamentary composite is 4.56×10^6 psi. This means that increase in fiber volume fraction by 0.4 (from 0.2 to 0.6), the increase in the modulus is about 11.4 times that of the matrix for glass-epoxy system when the fibers are continuous and they are in the direction of loading. But the increase in the modulus of randomly oriented filamentary composites for the same increase in fiber volume fraction is only 1.27×10^5 psi which is about 0.3 times the matrix modulus. Obviously this does not tally with the prediction of Tsai and Pagano[11] which predicts almost an order of magnitude higher increase. Detailed experimental verification alone can throw some light on this prediction. However, the artificial way of increasing the fiber volume fraction might be a possible reason for the discrepancy.

5.3.4 Residual Stresses

It is reported that the residual stresses donot have any severe effect on the uniaxial strength of cross-ply composites.[13] But no literature is available about the influence of residual stresses on randomly oriented filamentary composites. Particularly this might have significant influence from the viewpoint of

fracture mechanics. So, the study is carried out for this parametric effect too. The same test specimen, shown in fig. 8a, is studied under uniform tension when the reinforcing bars are subjected to varying degrees of initial stress. The initial stresses are all chosen as compressive and four values are chosen, namely, -250 psi, -750 psi, -1000 psi and -2000 psi. Since the initial stresses are assumed to be introduced due to differential thermal expansions, they are introduced in all the bars irrespective of their orientations.

Interestingly, even with such a wide variation in residual stresses, the Young's modulus of the test specimen remains almost the same. The value is about 9.4×10^5 psi in all the cases. However, there is a very slow but definite tendency of reduction in the maximum stress concentration factor with increasing initial stress of compressive type. Table 6 shows the summary of the analyses. It is noted that the reduction in stress concentration is very small compared to the increase in initial stress. The maximum normalised stresses, σ_x and σ_{xy} increase almost by the same amount. On one hand, reduction in maximum stress concentration is helpful, even if the reduction is small. On the other hand, increase in maximum shear stress is detrimental, particularly because shear plays a very important role for the composites, in general. This means that inadvertently introduced residual stresses does not help the fabricator in the ultimate analysis. Also deliberate introduction of residual stresses will not equally help, as is evidenced from the trend shown in table 6.

5.4 Conclusion:

The methodology established earlier for the analysis of randomly oriented filamentary composites by computer simulation technique, adds the reliability and opens up the potentiality for parametric study. The convergence of the non-homogeneous structure to the isotropic system is really interesting, particularly when the number of randomly oriented fibers are small. From the parametric study it appears that the effect of residual stress can be neglected for all practical purposes for this group of materials too. It adds no advantage to designer even in terms of fracture mechanics. It appears from the analysis that the attempt to increase the Young's modulus by using fibers of larger diameter has little success. The rate of increment of the modulus is very poor compared to the increase in fiber volume fraction. However, the modulus can be very readily increased if fibers are aligned in the direction of loading. A slight alignment improves the modulus to a considerable extent and the rate of increment follows a definite pattern.

It may be noted in conclusion that the shear modulus also follows the same trend under the influence of the above parameters.

CHAPTER VI

CONCLUSIONS

6.1 Summing-up:

Randomly oriented filamentary composites form an important group of composite materials. A methodology is presented here to make the microstructural analysis and study the macroscopic response of these materials by computer simulation technique. It is found that the random fibers give rise to much higher stress concentration factor in the specimen than that if the fibers are regularly spaced. However, shear does not play any additional detrimental role in this case. The interesting feature revealed in the analysis is the in-plane bending effect. The local inhomogeneities in the simulated test specimen give rise to such bending effect. But in actual system the random fibers are thousands in number unlike two, chosen in the transverse direction of the idealised specimen. This may smoothen out the local inhomogeneities and the bending effect may disappear. This needs to be treated in further detail if a better computer with more memory than the one used is available. At the same time careful experimental investigation may also throw some light. The microscopic analysis presented here shall always give a better insight about the performance of the random structure. It is also noted that the same analysis technique can be equally

extended without any additional trouble for regularly spaced or continuously reinforced composites.

In the macroscopic responses, under the influences of different parametric effects, it is noted that compressive residual stress has no effect and increased fiber volume fraction due to increased cross-sectional area has little effect on the Young's modulus of the composite. But biasness has got a very significant influence on the modulus. So a little success in aligning the fibers in the direction of loading will enhance the modulus considerably.

6.2 Recommendations:

The success of the analysis can be still improved by incorporating the following simple modifications for future investigations.

In the finite element analysis the hybrid method gives a definitely better estimate of the stress distribution than that given by the displacement, rather constant stress triangle, method. Similarly it can be seen if mixed method gives any better estimate. In this connection, it might be noted that in the mixed method both displacement compatibility and stress continuity is maintained between different phases of the composite which are supposed to play an important role for this group of materials. In hybrid method, however, stress continuity is sacrified to satisfy the equilibrium equations. In addition, instead of using constant stress bar elements as fibers, a

better bar elements can be used; particularly the one along which shear is allowed and/or variation of normal of stress is incorporated.

Only with two fiber orientation, convergence is achieved in the analysis. But the recommended practice is to use four orientations. So it will be of interest to see the changes, if any, in the results when four fiber orientations like 0°, 45°, 90° and 135° are used. Obviously this will require triangular elements implying larger memory.

Finally the methodology can be equally extended for evaluating the isotropic properties of randomly stacked laminated composite provided suitable changes are incorporated in the program.

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APPENDIX - A

The following are the matrices used in deriving the stiffness matrix of five term hybrid rectangular elements:

$$[P] = \begin{bmatrix} 1 & y & 0 & 0 & 0 \\ 0 & 0 & 1 & x & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[R] = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & -x & 0 \\ 1 & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & x & 0 \\ -1 & -y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$[N] = \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix}$$

$$[T] = \begin{bmatrix} -\frac{b}{2} & 0 & \frac{b}{2} & 0 & \frac{b}{2} & 0 & -\frac{b}{2} & 0 \\ -\frac{b^2}{6} & 0 & \frac{b^2}{6} & 0 & \frac{b^2}{3} & 0 & -\frac{b^2}{3} & 0 \\ 0 & -\frac{a}{2} & 0 & -\frac{a}{2} & 0 & \frac{a}{2} & 0 & \frac{a}{2} \\ 0 & -\frac{a^2}{6} & 0 & -\frac{a^2}{3} & 0 & \frac{a^2}{3} & 0 & \frac{a^2}{6} \\ -\frac{a}{2} & -\frac{b}{2} & -\frac{a}{2} & \frac{b}{2} & \frac{a}{2} & \frac{b}{2} & \frac{a}{2} & -\frac{b}{2} \end{bmatrix}$$

$$[L] = \begin{bmatrix} 1-\frac{x}{a} & 0 & \frac{x}{a} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\frac{x}{a} & 0 & \frac{x}{a} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-\frac{y}{b} & 0 & \frac{y}{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\frac{y}{b} & 0 & \frac{y}{b} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{x}{a} & 0 & 1-\frac{x}{a} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{x}{a} & 0 & 1-\frac{x}{a} \\ 1-\frac{y}{b} & 0 & 0 & 0 & 0 & 0 & \frac{y}{b} & 0 \\ 0 & 1-\frac{y}{b} & 0 & 0 & 0 & 0 & 0 & \frac{y}{b} \end{bmatrix}$$

APPENDIX - B

Fig. 1c shows the assembly of a bar and a rectangular element. For the bar IJ which can take only axial stresses, the stiffness matrix will be of the following form. It is noted that the stiffness elements of the bar along 1 and 3 degrees of freedom are zero

$$[k]_{bar} = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k'_{22} & 0 & k'_{24} \\ 0 & 0 & 0 & 0 \\ 0 & k'_{42} & 0 & k'_{44} \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

But for the hybrid rectangular element IJKL, the stiffness elements exist for all the eight degrees of freedom. Consequently the stiffness matrix takes the form

$$[k]_{rect.} = \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & \cdots & 8 \end{array} \\ \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & \cdots & k_{18} \\ k_{21} & k_{22} & k_{23} & k_{24} & \cdots & k_{28} \\ k_{31} & k_{32} & k_{33} & k_{34} & \cdots & k_{38} \\ k_{41} & k_{42} & k_{43} & k_{44} & \cdots & k_{48} \\ \vdots & & & & & \vdots \\ k_{81} & k_{82} & k_{83} & k_{84} & \cdots & k_{88} \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ \vdots \\ 8 \end{array}$$

In order to assemble, the elements of the stiffness matrix of identical degrees of freedom are to be identified and then added to get the overall stiffness matrix of the assembled structure. So for fig. 1c, the overall stiffness matrix becomes:

$$[K] = \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & \cdots & 8 \end{array} \\ \left[\begin{array}{ccccccc} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & \cdots & k_{18} \\ k_{21} & (k_{22} + k'_{22}) & k_{23} & (k_{24} + k'_{24}) & k_{25} & \cdots & k_{28} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & \cdots & k_{38} \\ k_{41} & (k_{42} + k'_{42}) & k_{43} & (k_{44} + k'_{44}) & k_{45} & \cdots & k_{48} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & \cdots & k_{58} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & \cdots & k_{68} \\ k_{71} & k_{72} & k_{73} & k_{74} & k_{75} & \cdots & k_{78} \\ k_{81} & k_{82} & k_{83} & k_{84} & k_{85} & \cdots & k_{88} \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

Similarly a horizontal bar element like JK can also be assembled with the corresponding addition of stiffness elements along the common degrees of freedom of 3 and 5. In the program this assembly process is done by the computer.

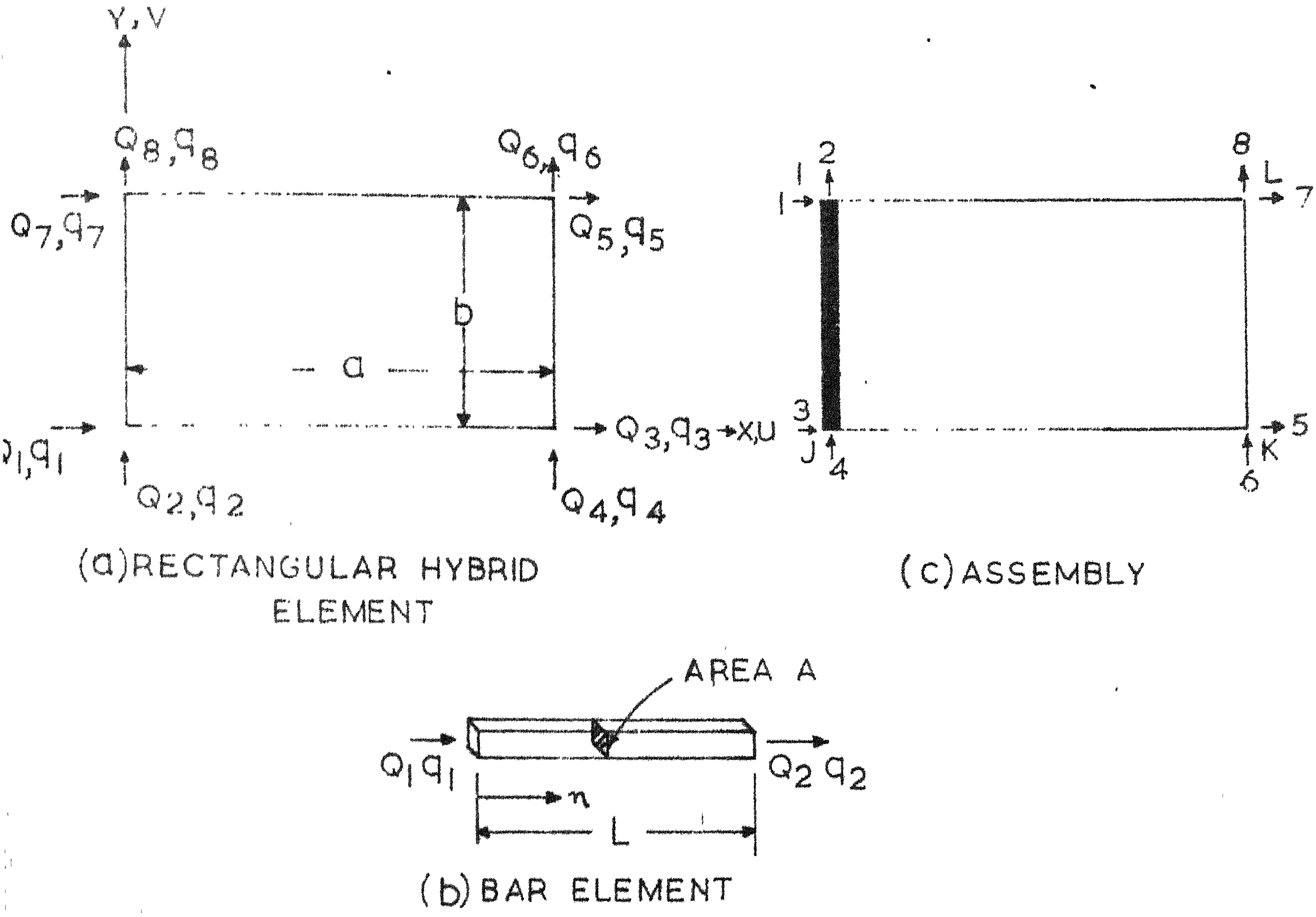


FIG. 1 GENERALISED FORCES & DISPLACEMENTS OF a PIAN'S RECTANGULAR HYBRID ELEMENT b CONSTANT STRESS BAR ELEMENT & c THEIR ASSEMBLY.

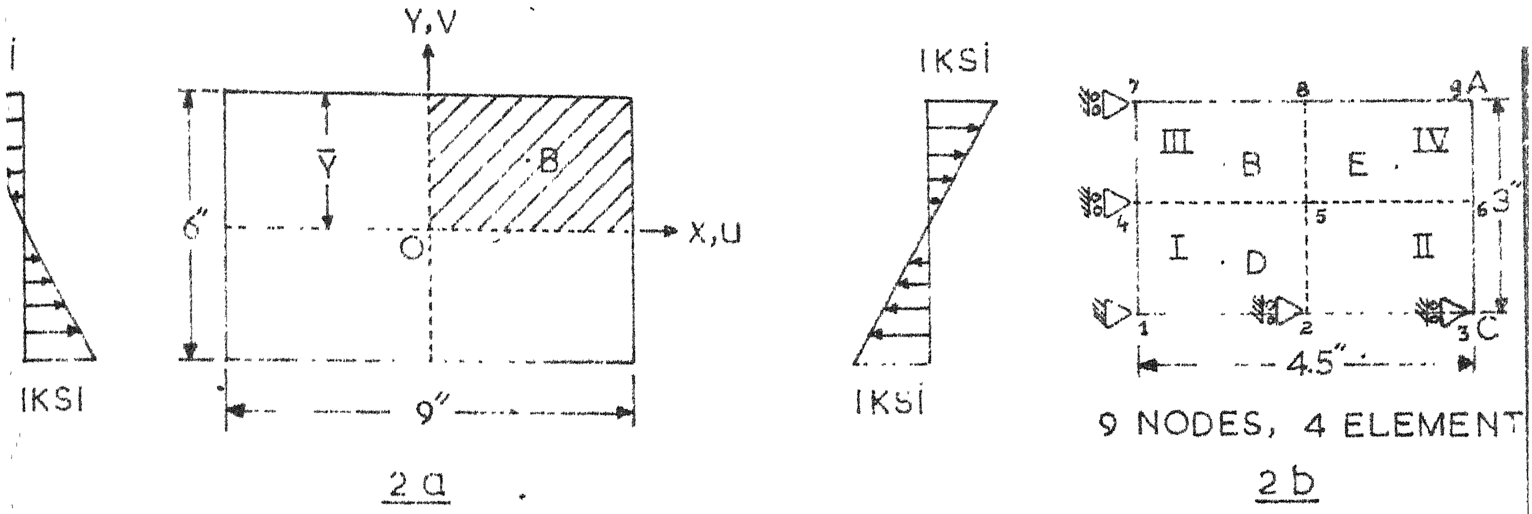


FIG 2a BEAM UNDER PURE BENDING & b CORRESPONDING FINITE ELEMENT MESH

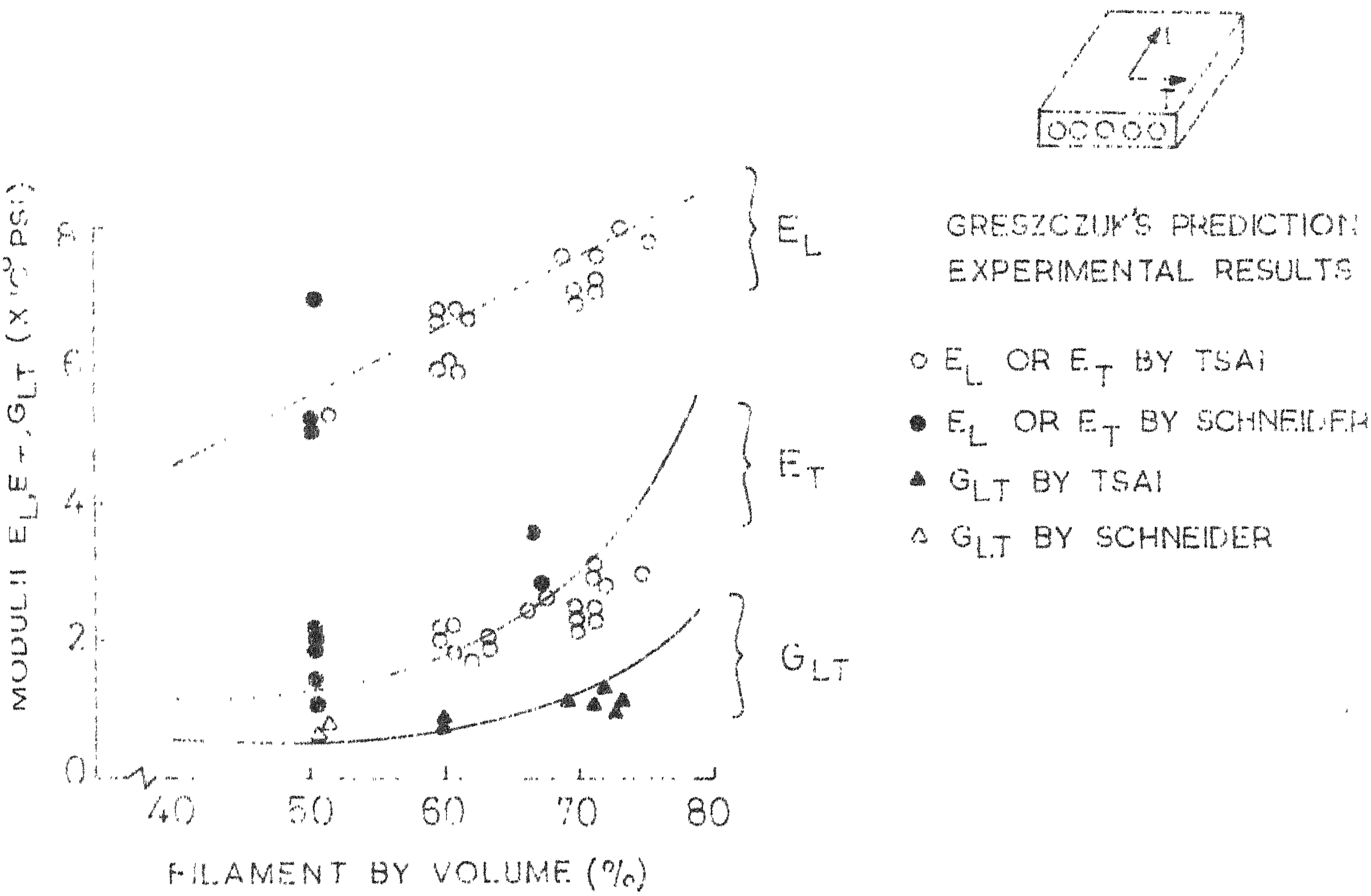
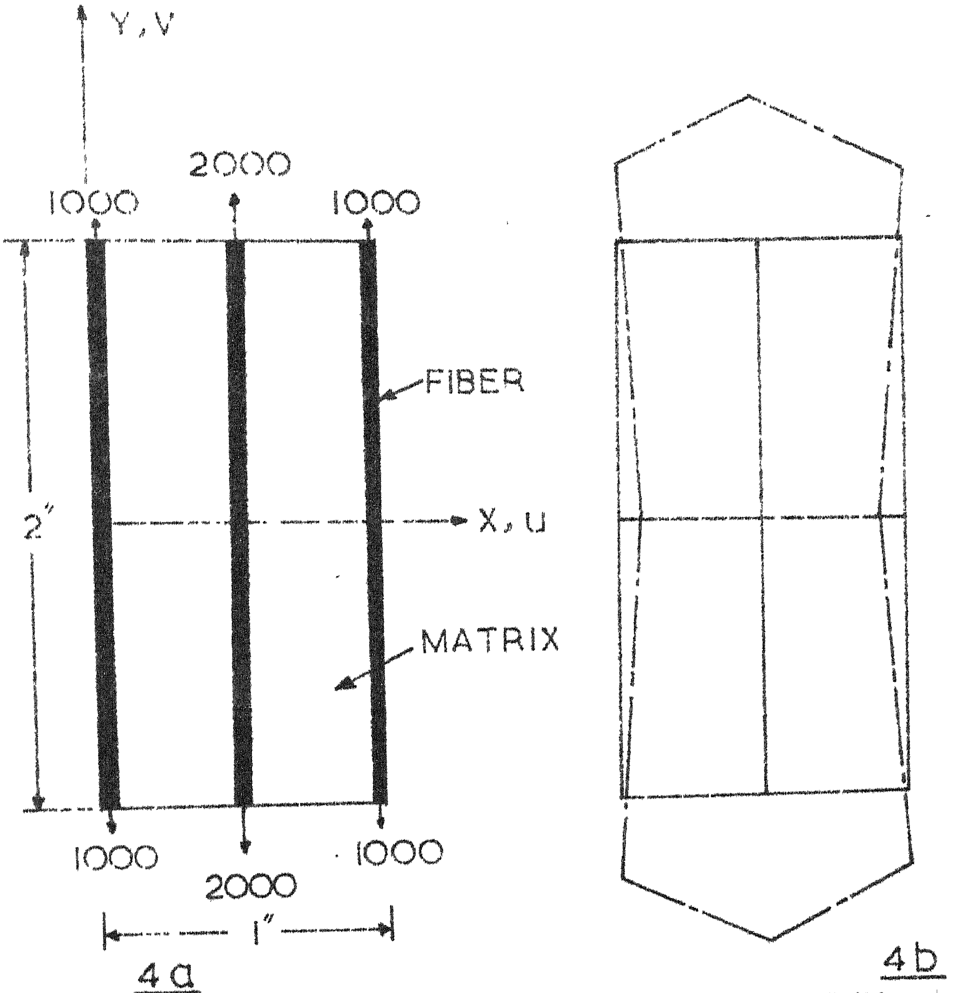


FIG. 3 EXPERIMENTAL RESULTS & GRESZCZUK'S PREDICTION OF PRINCIPAL ELASTIC PROPERTIES OF E-GLASS EPOXY COMPOSITES



CROSS-SECTION OF FIBER = 0.2 SQ IN

MATERIAL PROPERTIES

$E_f = 10.6 \times 10^6$ PSI

$\nu_f = 0.22$

$E_m = 0.5 \times 10^6$ PSI

$\nu_m = 0.35$

FIBER VOLUME FRACTION = 0.6

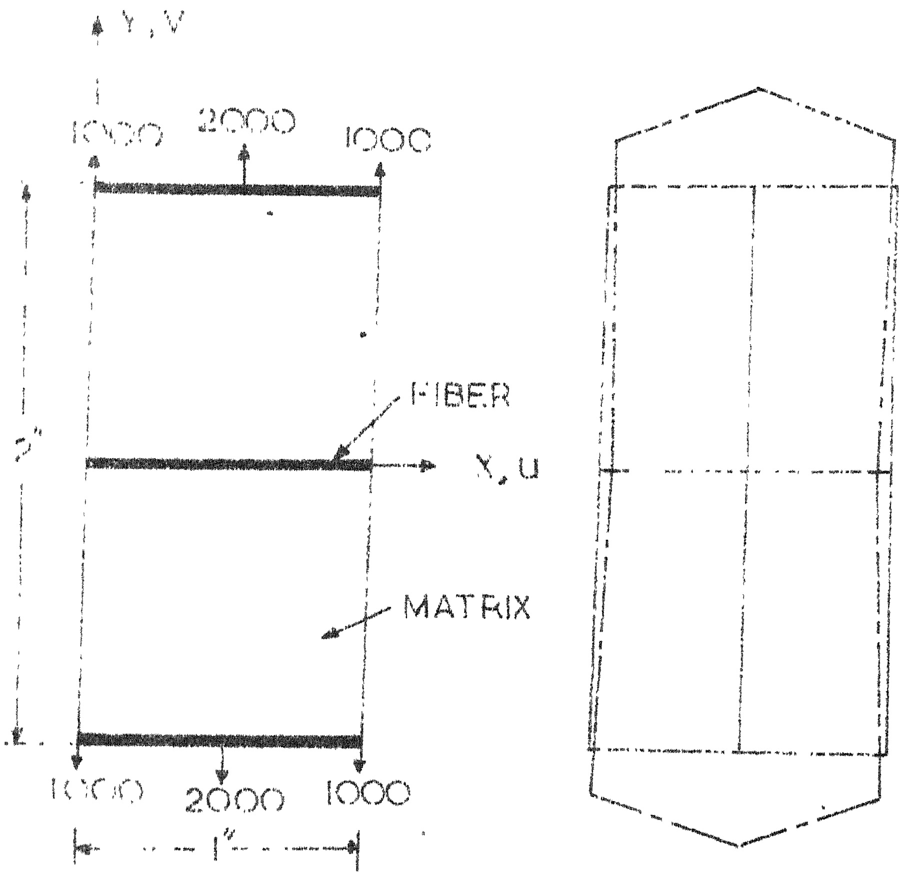
SPECIMEN SIZE 2"X 1"X 1"

IN FIG. 4b

— : FINITE ELEMENT DIVISIONS

---- : DEFORMATION PATTERN

FIG. 4a CONTINUOUSLY REINFORCED COMPOSITE WITH LONGITUDINAL FIBERS & b ITS DEFORMATION PATTERN.



5a   5b

CROSS SECTION OF FIBERS = 0.2 SQ. IN

MATERIAL PROPERTIES

$E_f = 10.6 \times 10^6$ PSI

$\nu_f = 0.22$

$E_m = 0.5 \times 10^6$ PSI

$\nu_m = 0.35$

FIBER VOLUME FRACTION = 0.3

SPECIMEN SIZE 2" X 1" X 1"

IN FIG. 5 b

—: FINITE ELEMENT DIVISIONS

---: DEFORMATION PATTERN

FIG. 5a CONTINUOUSLY REINFORCED COMPOSITE WITH LONGITUDINAL FIBERS & b ITS DEFORMATION PATTERN.

FIG 6 a

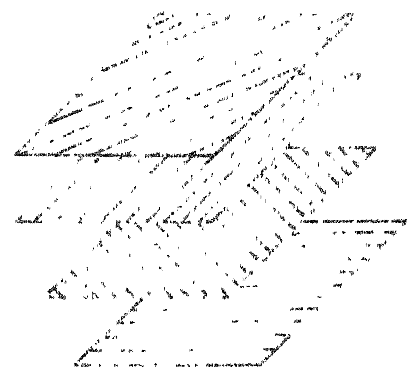


FIG 6 b

FIG 6 a RANDOMLY ORIENTED FILAMENTARY COMPOSITE PLATE & b MULTILAYERED COMPOSITE GIVING RISE TO ISOTROPIC PROPERTIES.

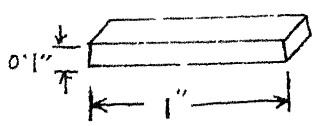


FIG 7a

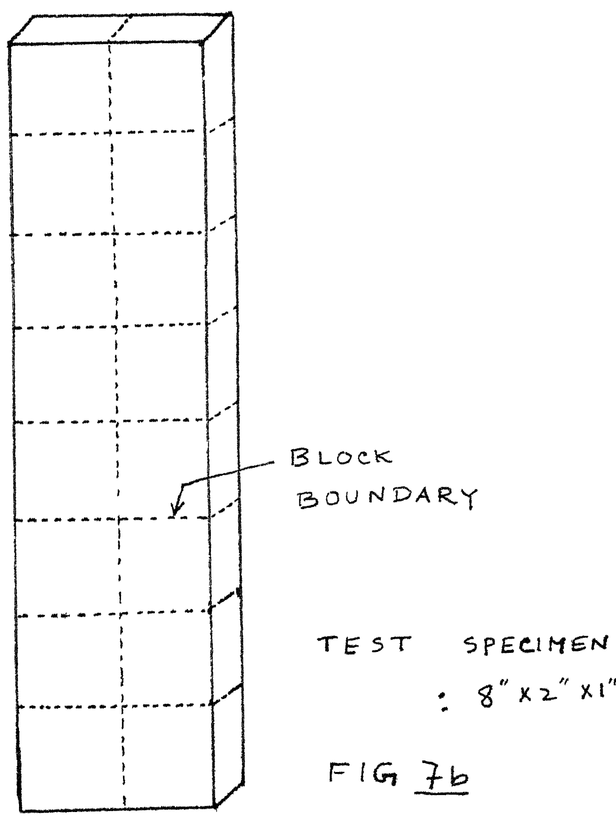


FIG 7b

FIG 7 a TYPICAL FIBER GEOMETRY & b THE MATRIX BLOCKS TO BE FILLED BY FIBERS.

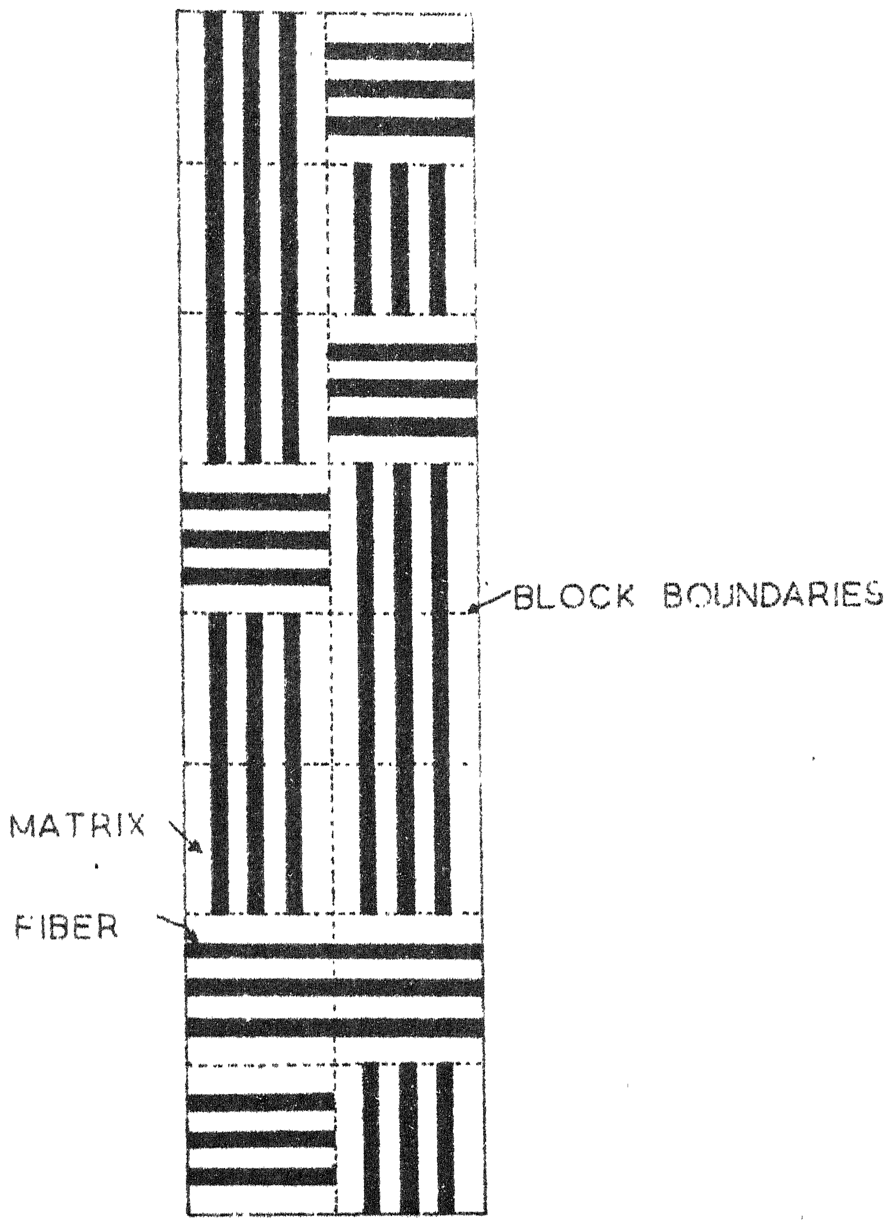
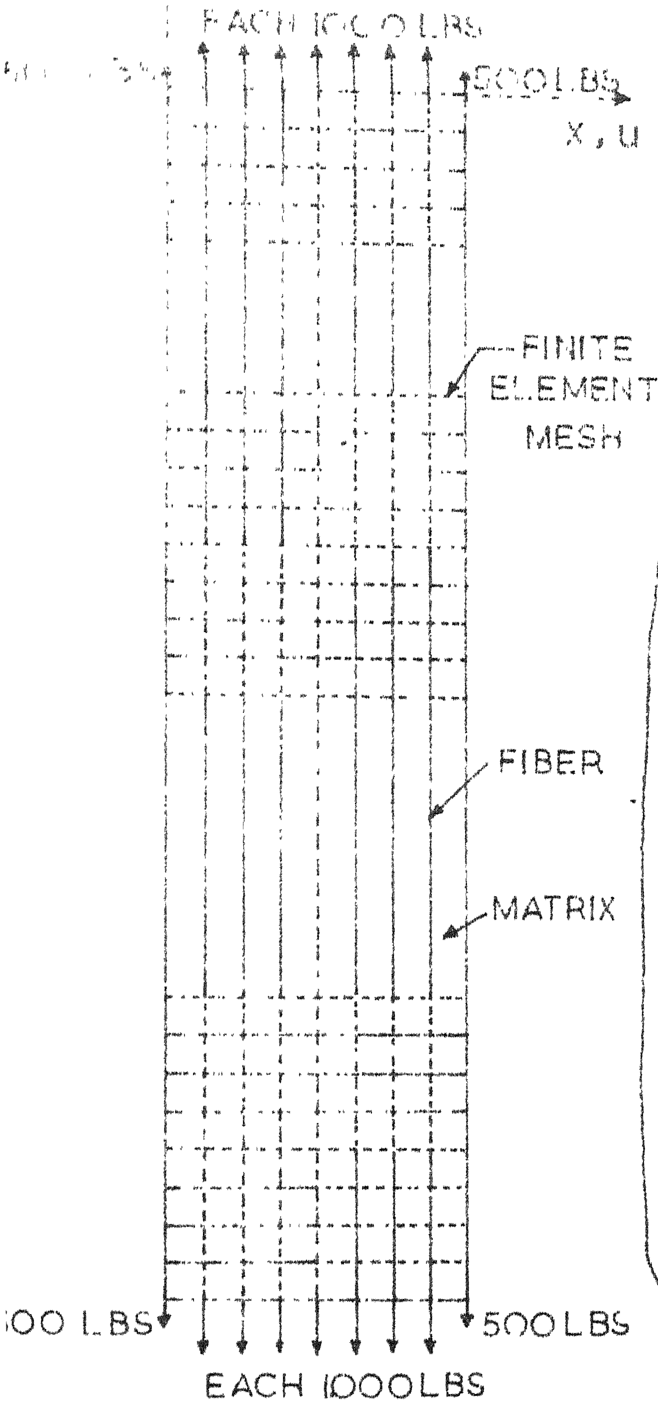
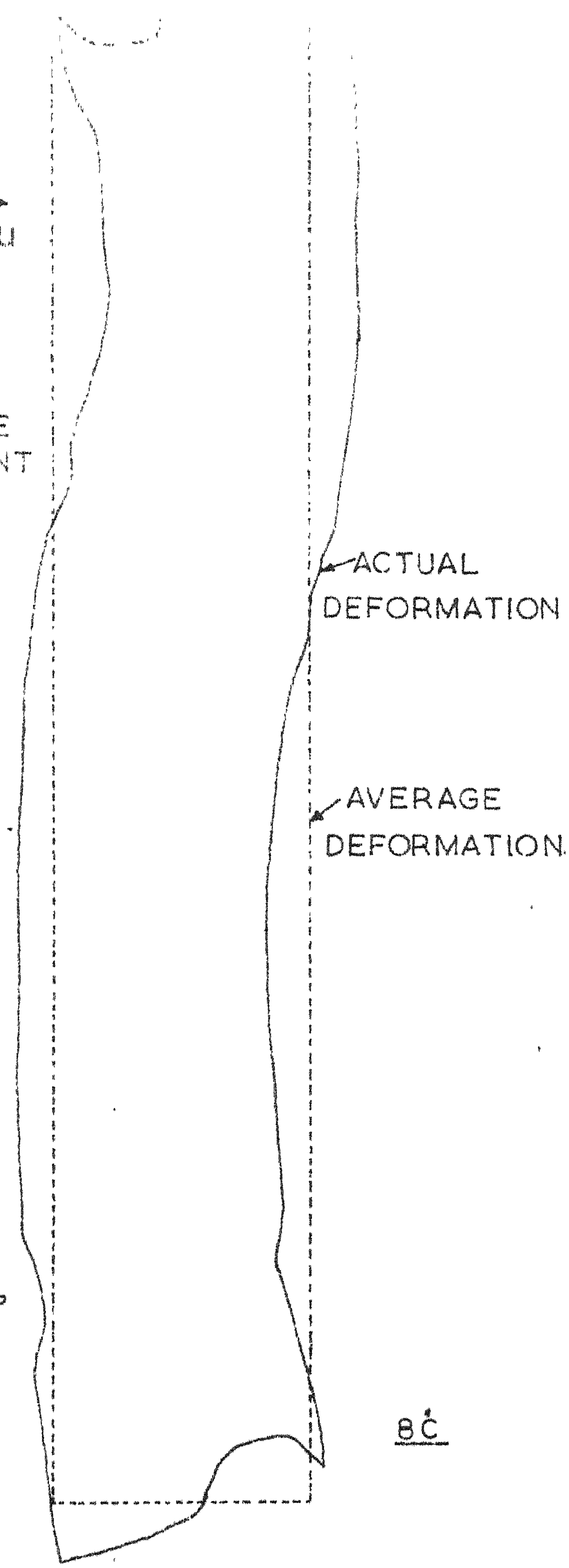


FIG. 8a THE TEST SPECIMEN: COMPUTER SIMULATION OF RANDOMLY ORIENTED FILAMENTARY COMPOSITE WITH V_f=30% DIMENSION 8"x 2"x 1"



NO. OF NODES=207
NO. OF LLEMENTS=308

8 b



8 c

FIG 8 b FINITE ELEMENT IDEALISATION OF RANDOMLY ORIENTED FILAMENTARY COMPOSITE TEST SPECIMEN.

FIG. 8 c DEFORMATION PATTERN (AVERAGE & ACTUAL) OF THE TEST SPECIMEN (DEFORMATION MAGNIFICATION=100)

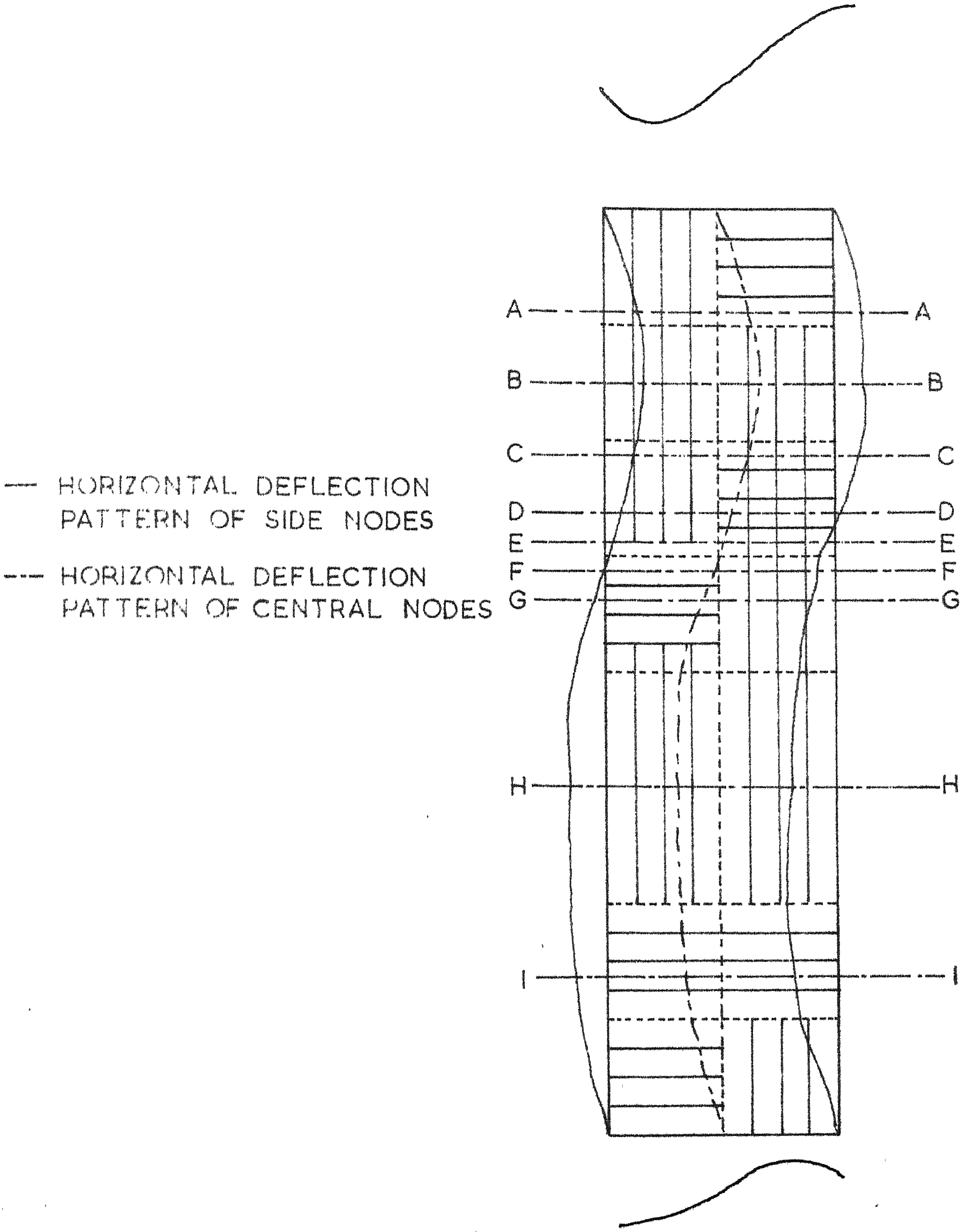
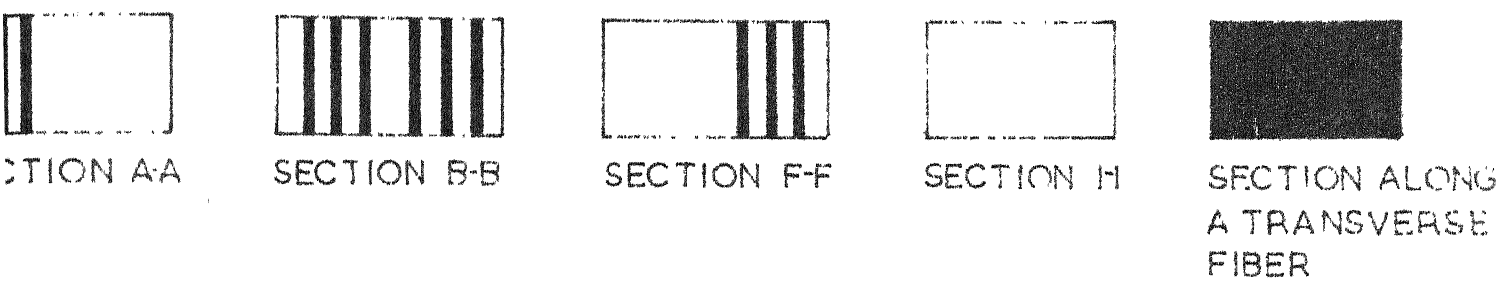


FIG. 9 SEPARATED BENDING DEFORMATION DUE TO COUPLING BETWEEN PURE STRETCHING & BENDING. ALSO SHOWN THE CROSS-SECTIONS USED LATER ON.



. 10 DIFFERENT CROSS-SECTIONS (2" x 1") OF THE TEST SPECIMEN.

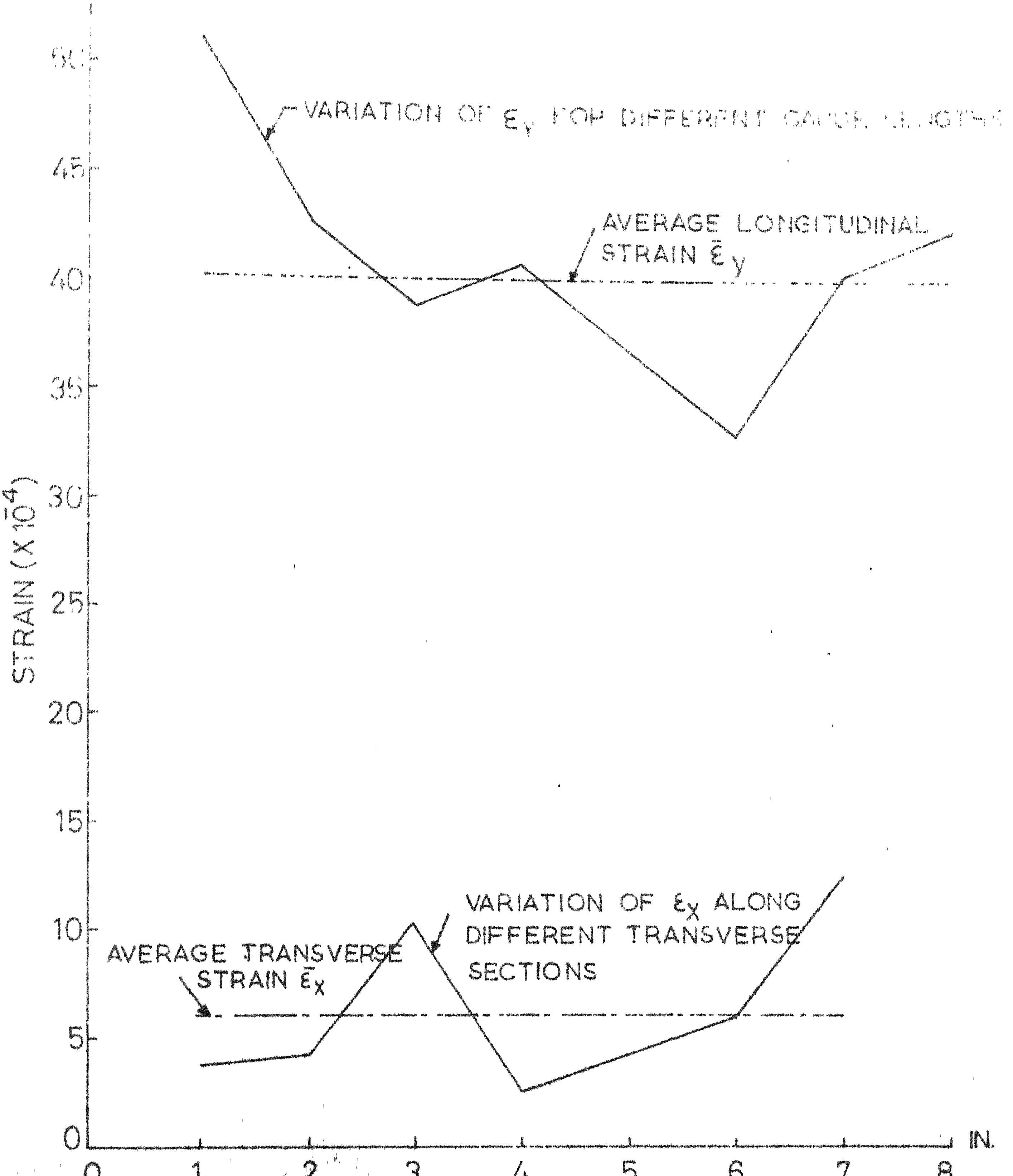


FIG. 11 VARIATION OF ε_x AND ε_y FOR THE TEST SPECIMEN

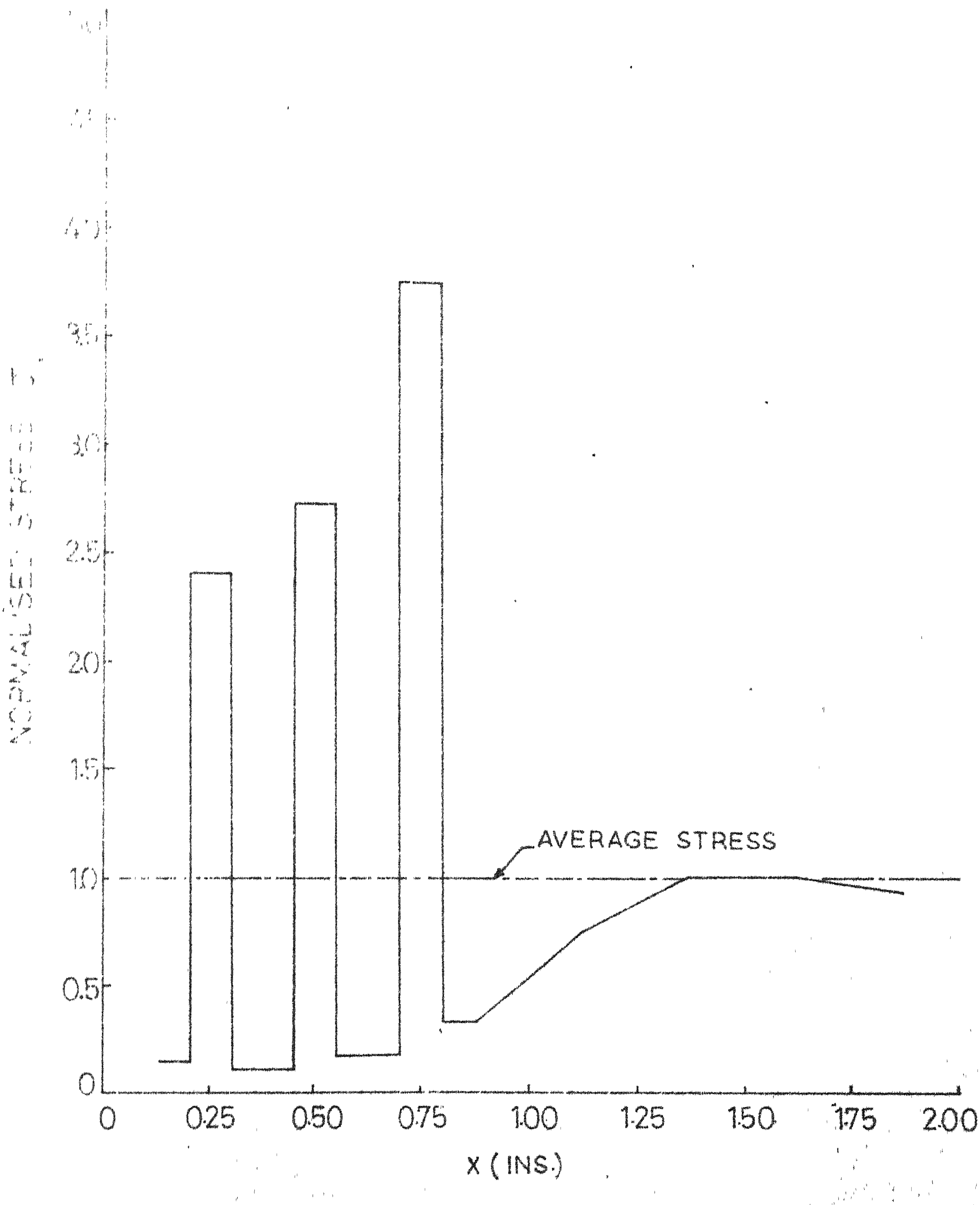


FIG. 12 VARIATION OF NORMALISED STRESS σ_y ALONG THE TRANSVERSE SECTION D-D

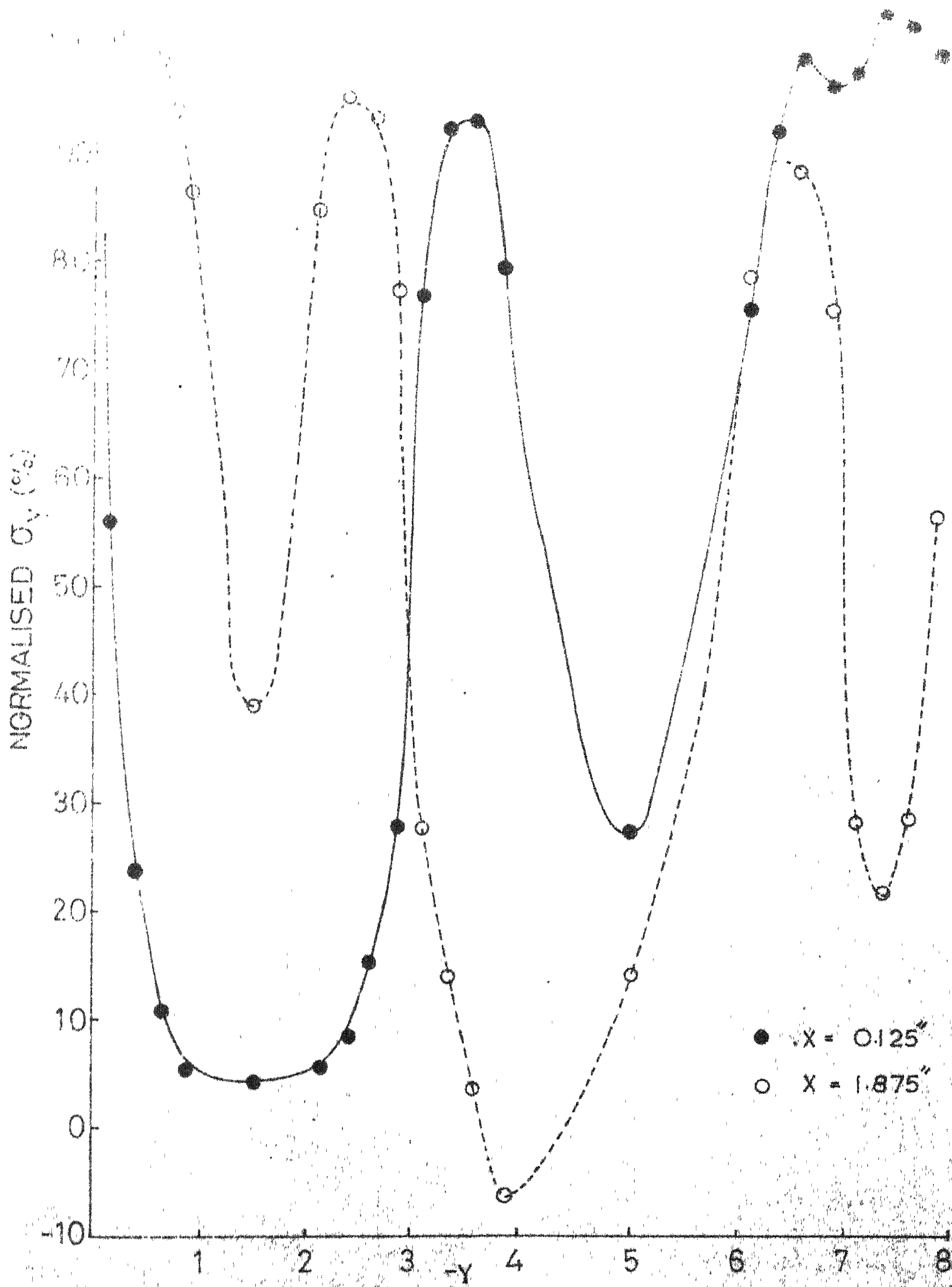


FIG. 13 VARIATION OF NORMALISED σ_y ALONG LONGITUDINAL CROSS-SECTIONS

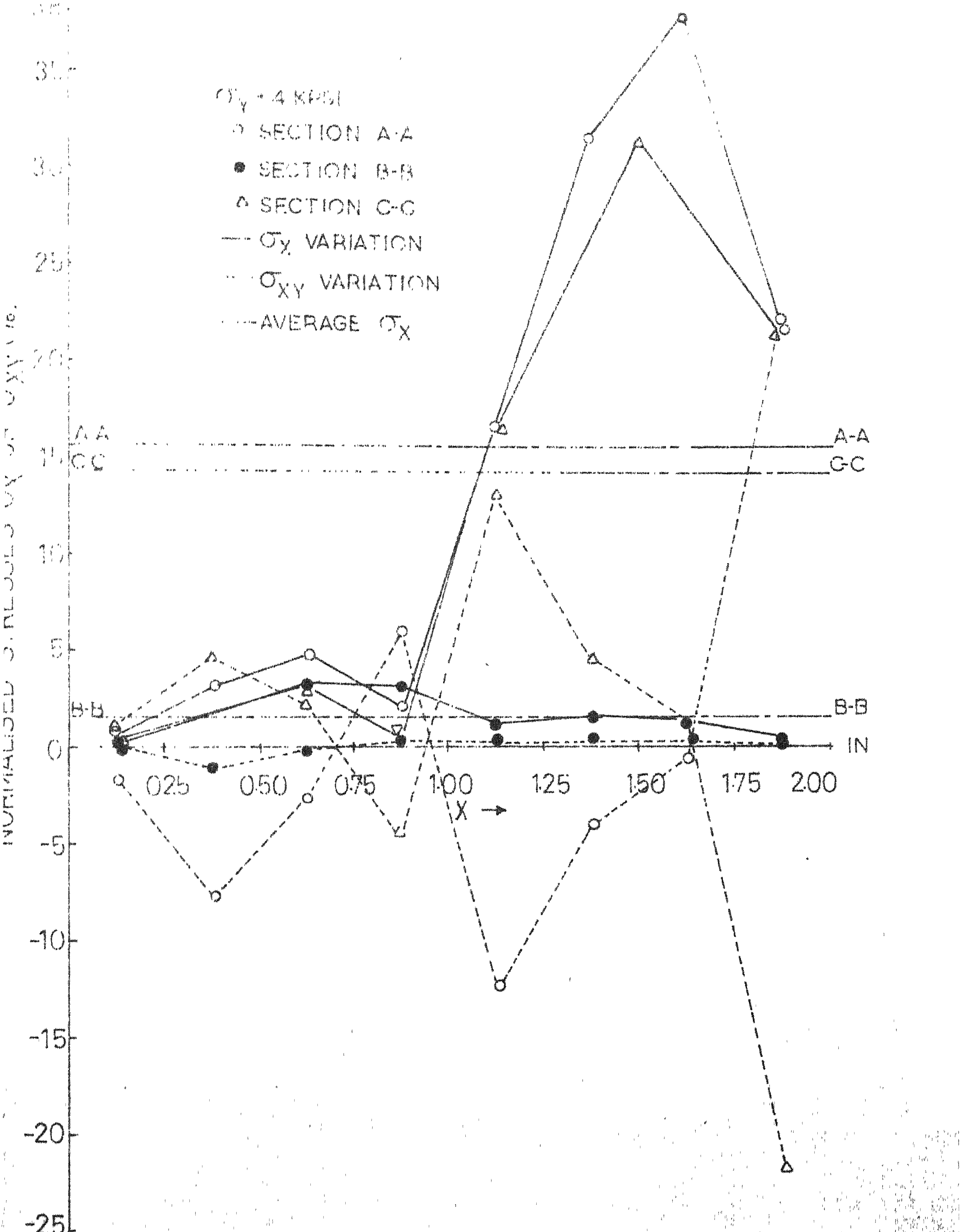


FIG. 14 DISTRIBUTION OF NORMALISED STRESSES AT DIFFERENT CROSS-SECTIONS SHOWN IN FIG. 9

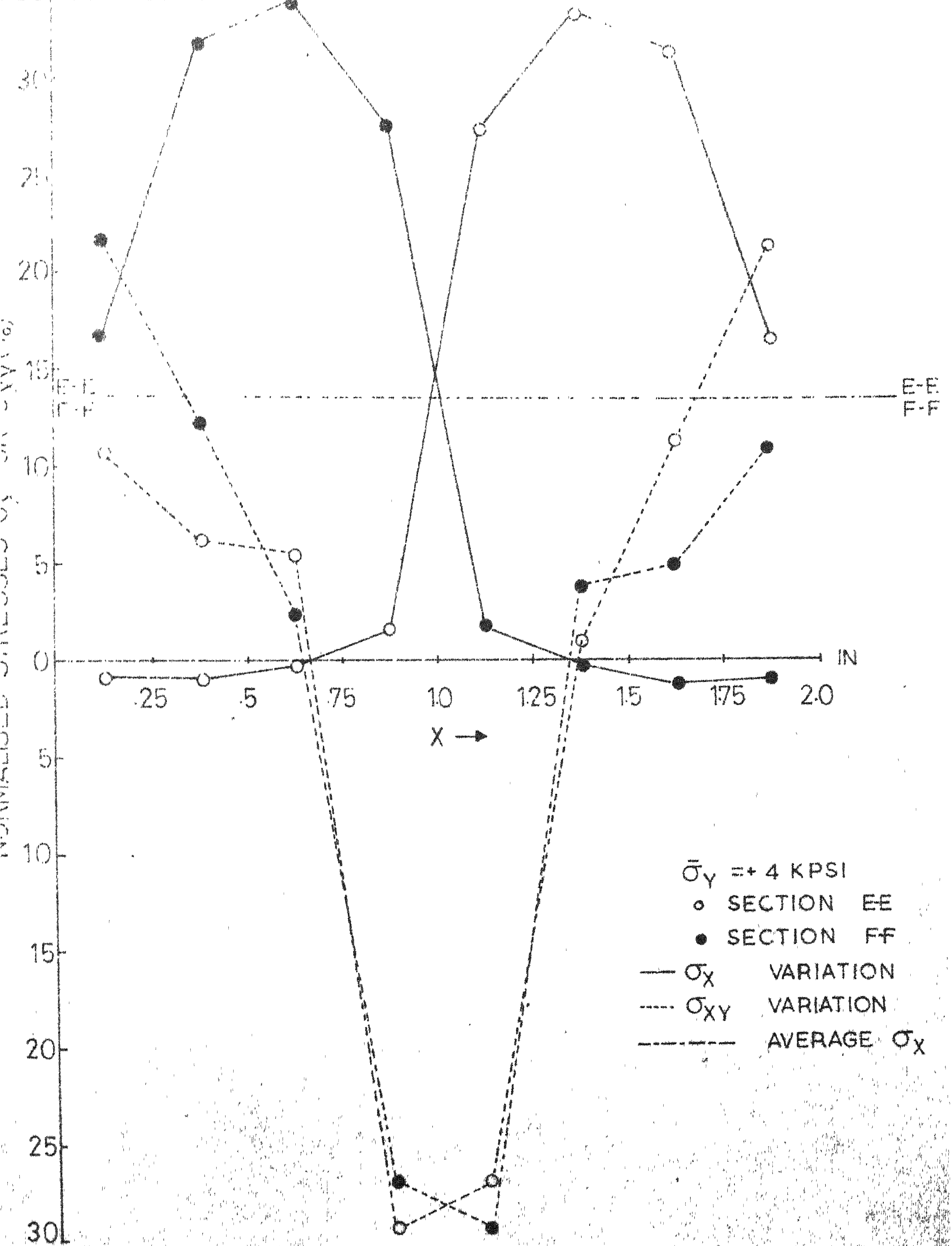


FIG. 15 DISTRIBUTION OF NORMALISED STRESSES AT DIFFERENT CROSS-SECTION SHOWN IN FIG. 9

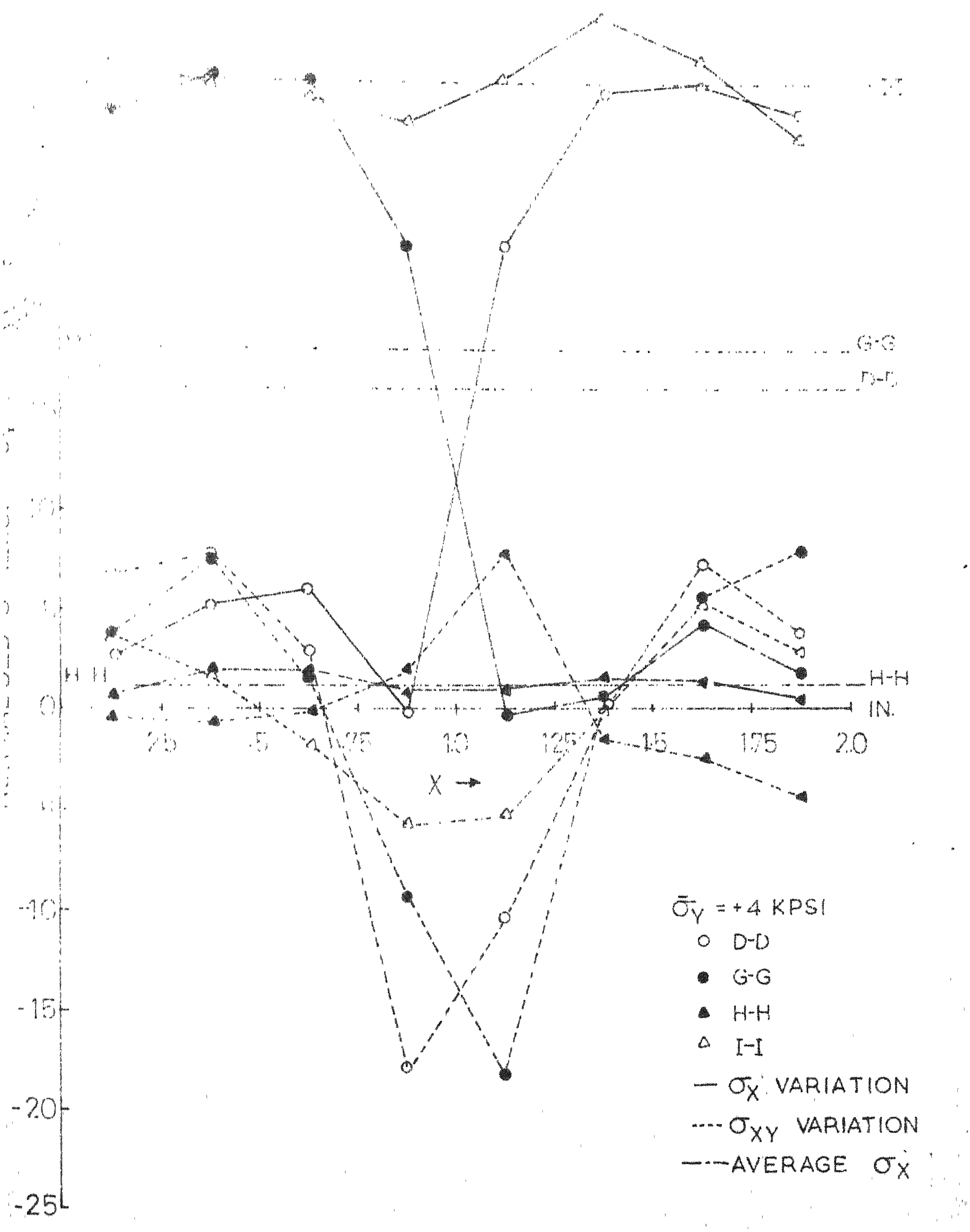


FIG. 16 DISTRIBUTION OF NORMALISED STRESSES AT DIFFERENT CROSS-SECTION SHOWN IN FIG. 9

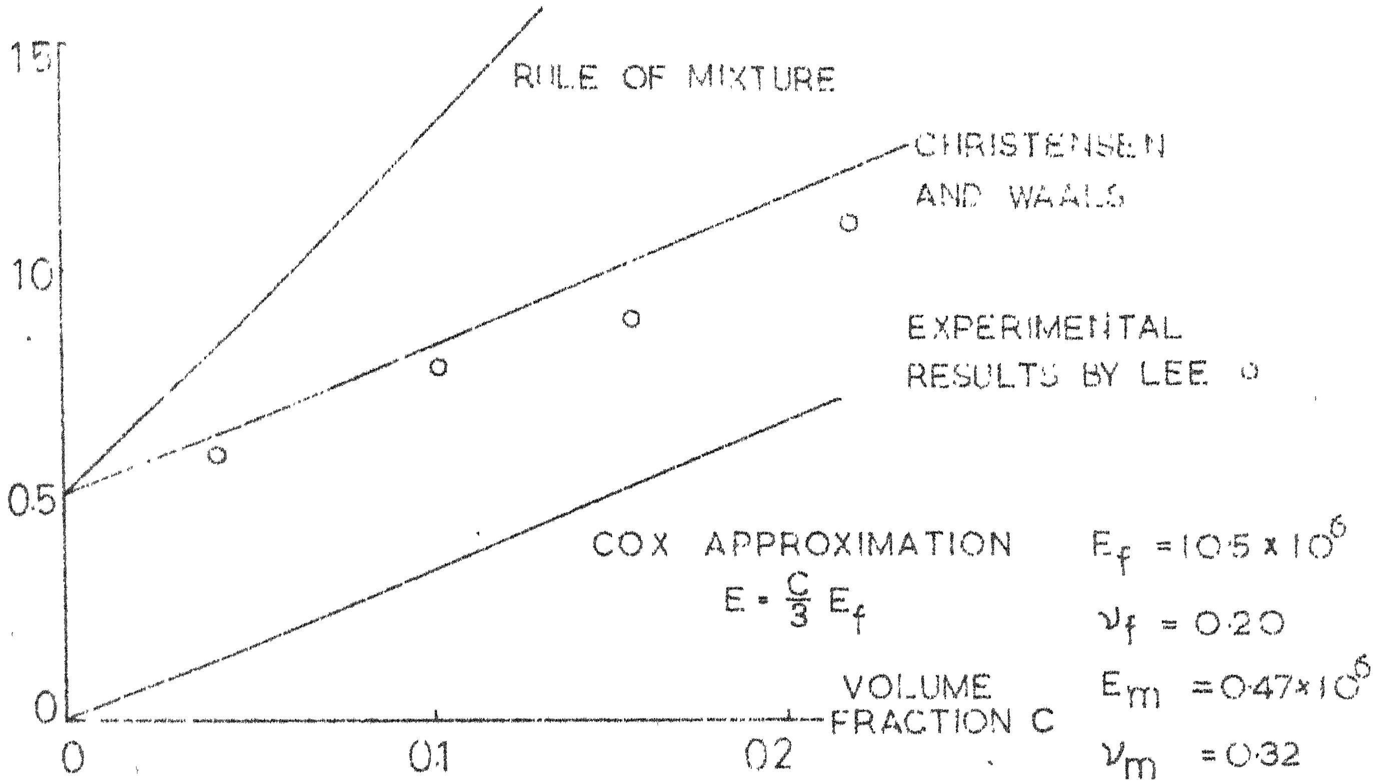


FIG. 17 COMPARISON OF EXPERIMENTAL RESULTS AND DIFFERENT PREDICTIONS FOR GLASS POLYSTYRENE SYSTEM.

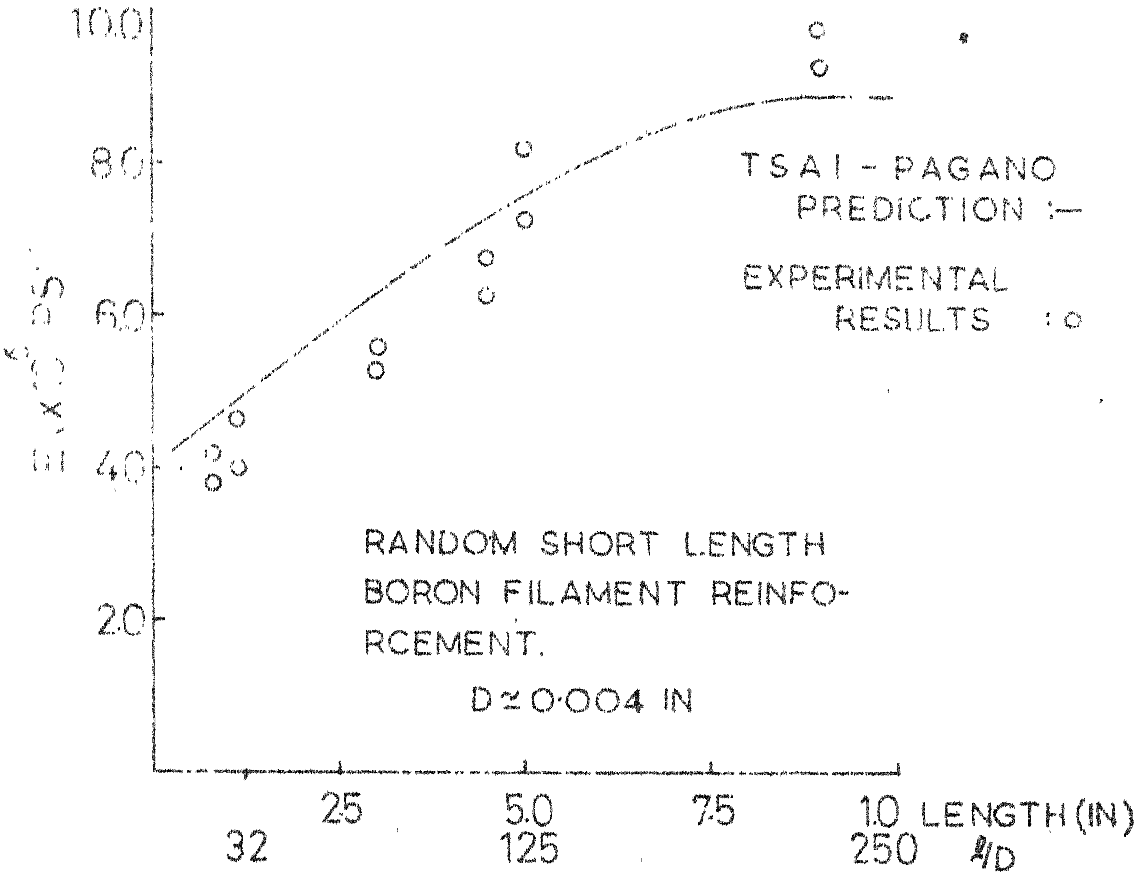
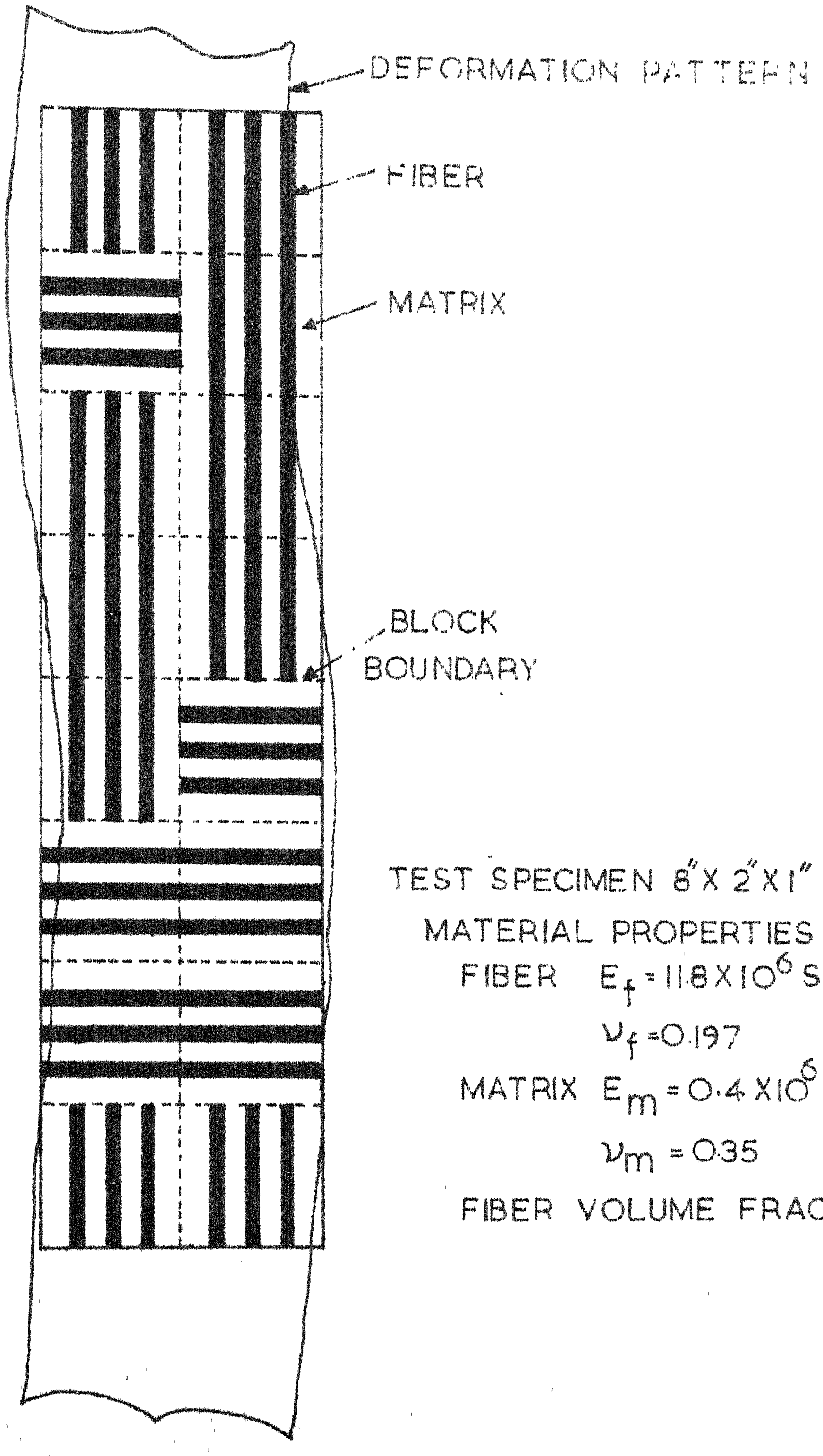


FIG. 18 COMPARISON OF EXPERIMENTAL RESULTS & TSAI-PAGANO PREDICTION FOR RANDOM BORON FIBER REINFORCED EPOXY COMPOSITE (V_f = 0·40)



TEST SPECIMEN 8" X 2" X 1"

MATERIAL PROPERTIES

FIBER $E_f = 11.8 \times 10^6$ SPI

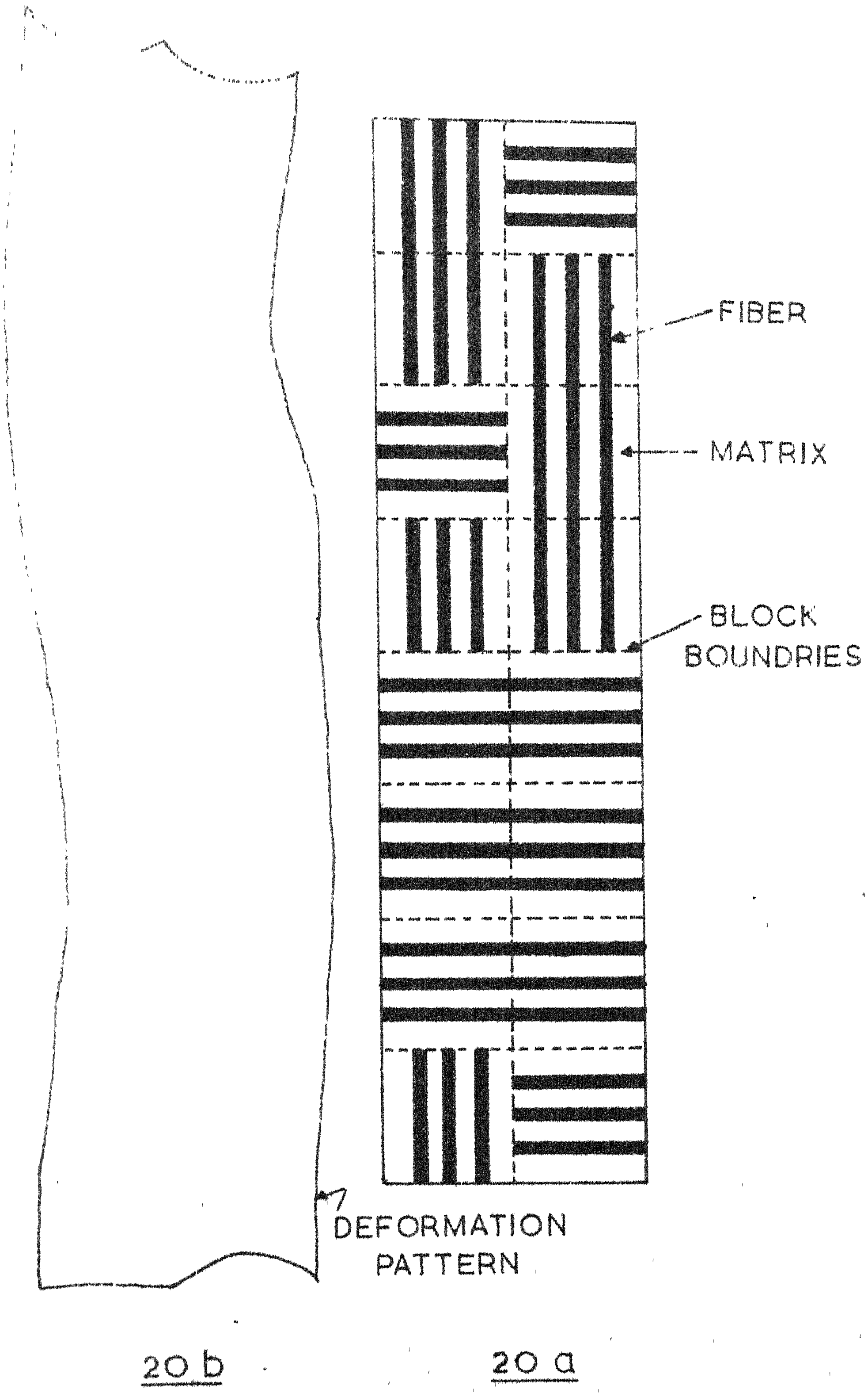
$\nu_f = 0.197$

MATRIX $E_m = 0.4 \times 10^6$ SPI

$\nu_m = 0.35$

FIBER VOLUME FRACTION = 0.3

FIG. 19 LONGITUDINALLY ARRANGED RANDOM FIBER TEST SPECIMEN & ITS DEFORMATION PATTERN UNDER UNIFORM TENSION.



IG. 20 a COMPOSITE TEST SPECIMEN WITH 50% BIASNESS &

b CORRESPONDING DEFORMATION PATTERN.

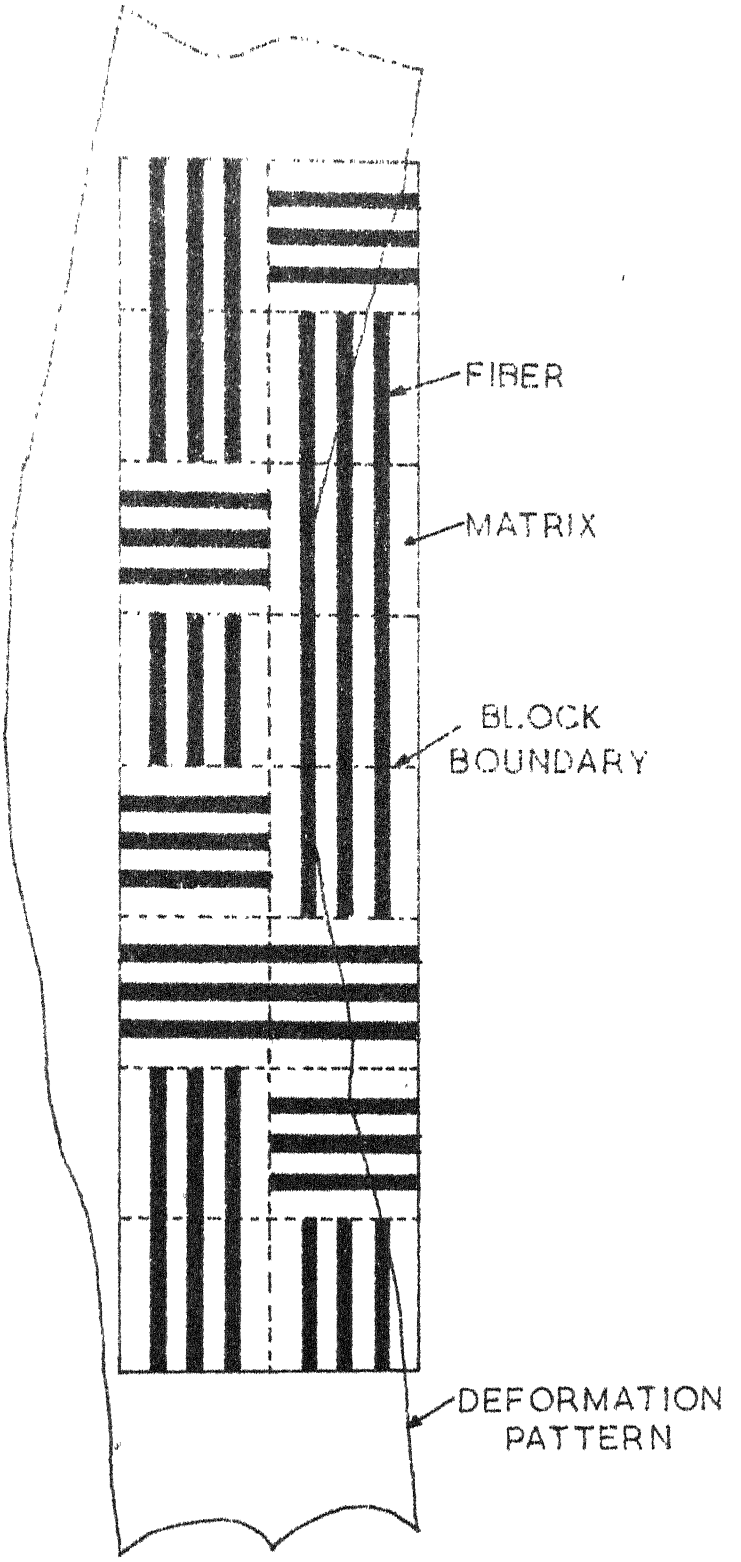


FIG. 21

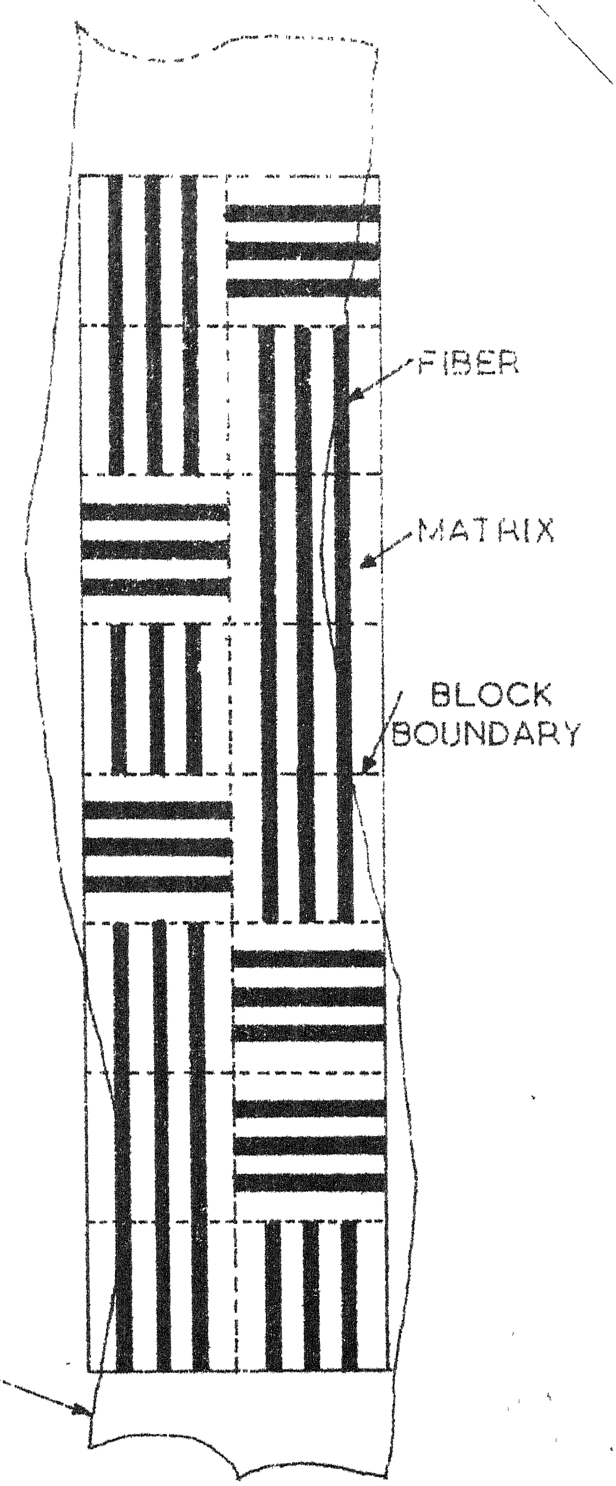


FIG 22

FIGS. 21-22 COMPOSITE TEST SPECIMEN WITH(21)60% &(22) 70% BIASNESS & CORRESPONDING DEFORMATION PATTERN.

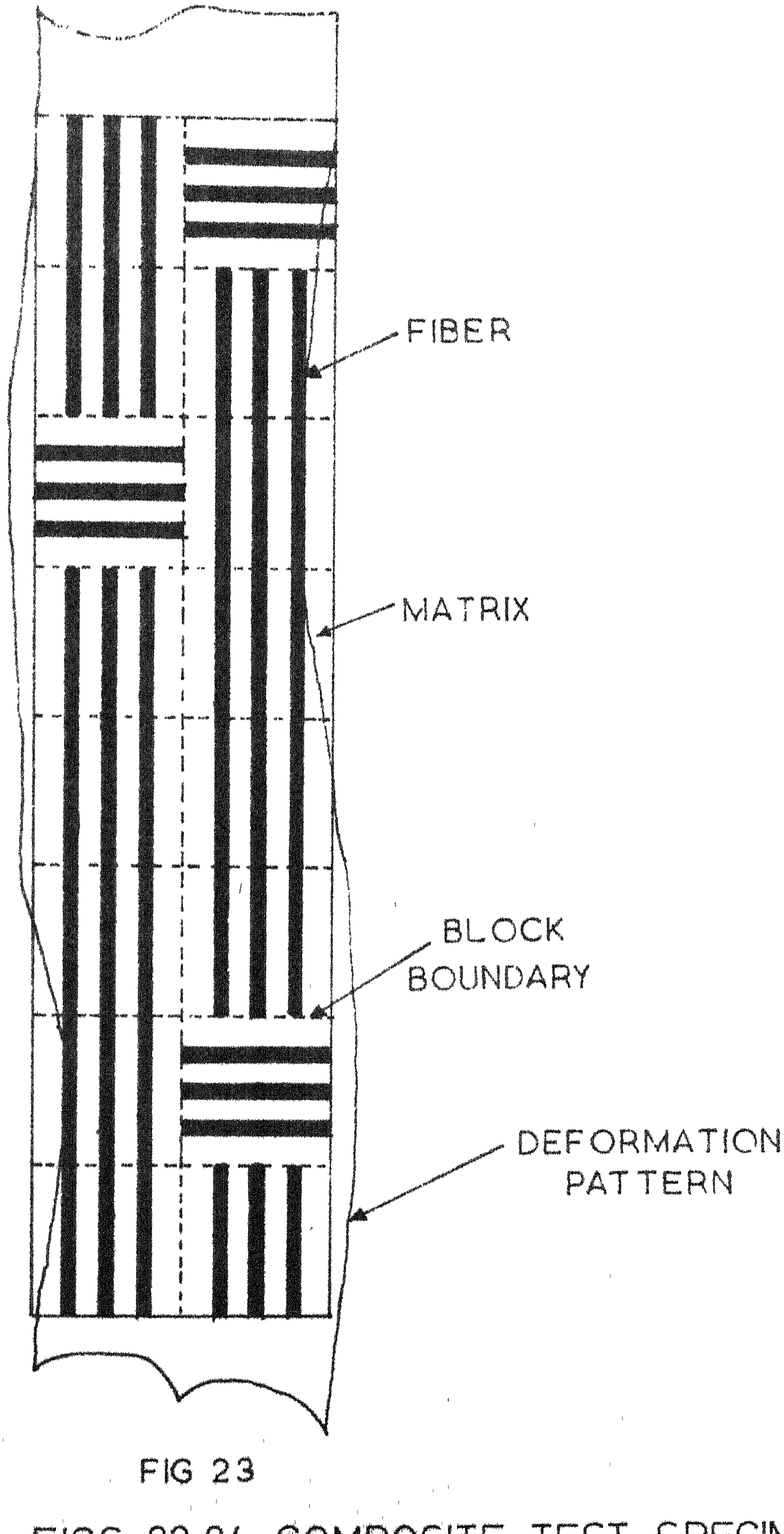


FIG 23

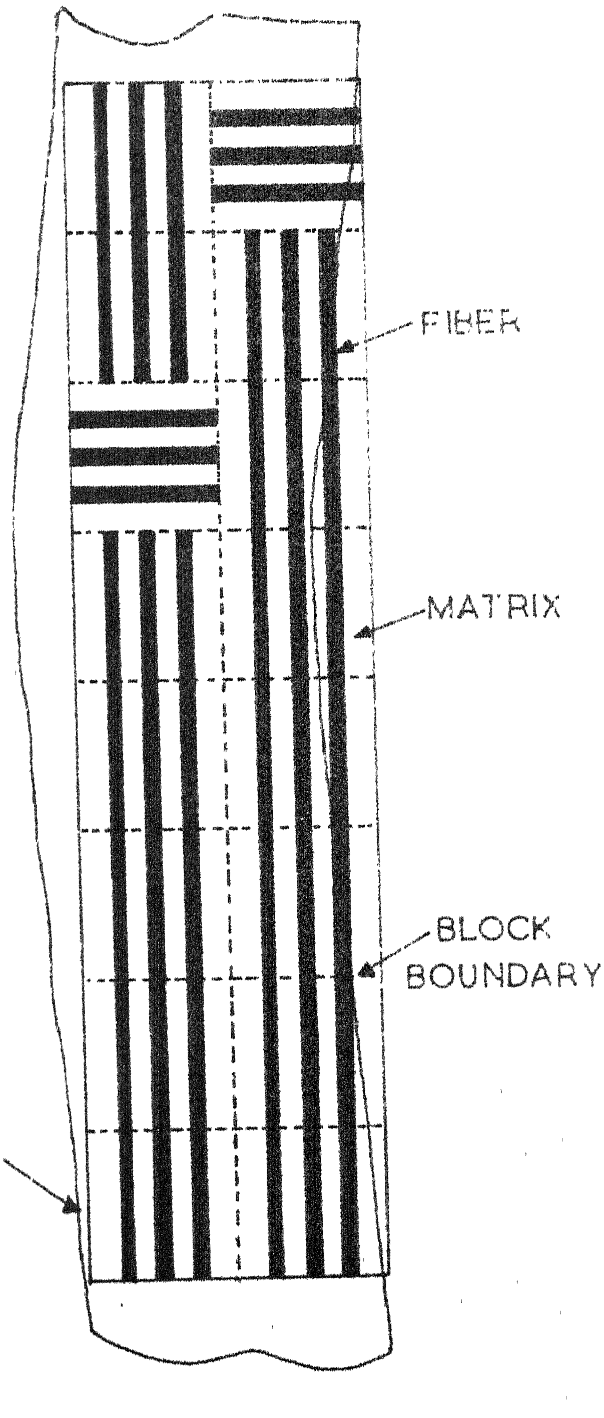


FIG 24

FIGS. 23-24 COMPOSITE TEST SPECIMEN WITH (23) 80% & (24) 90% BIASNESS & CORRESPONDING DEFORMATION PATTERN.

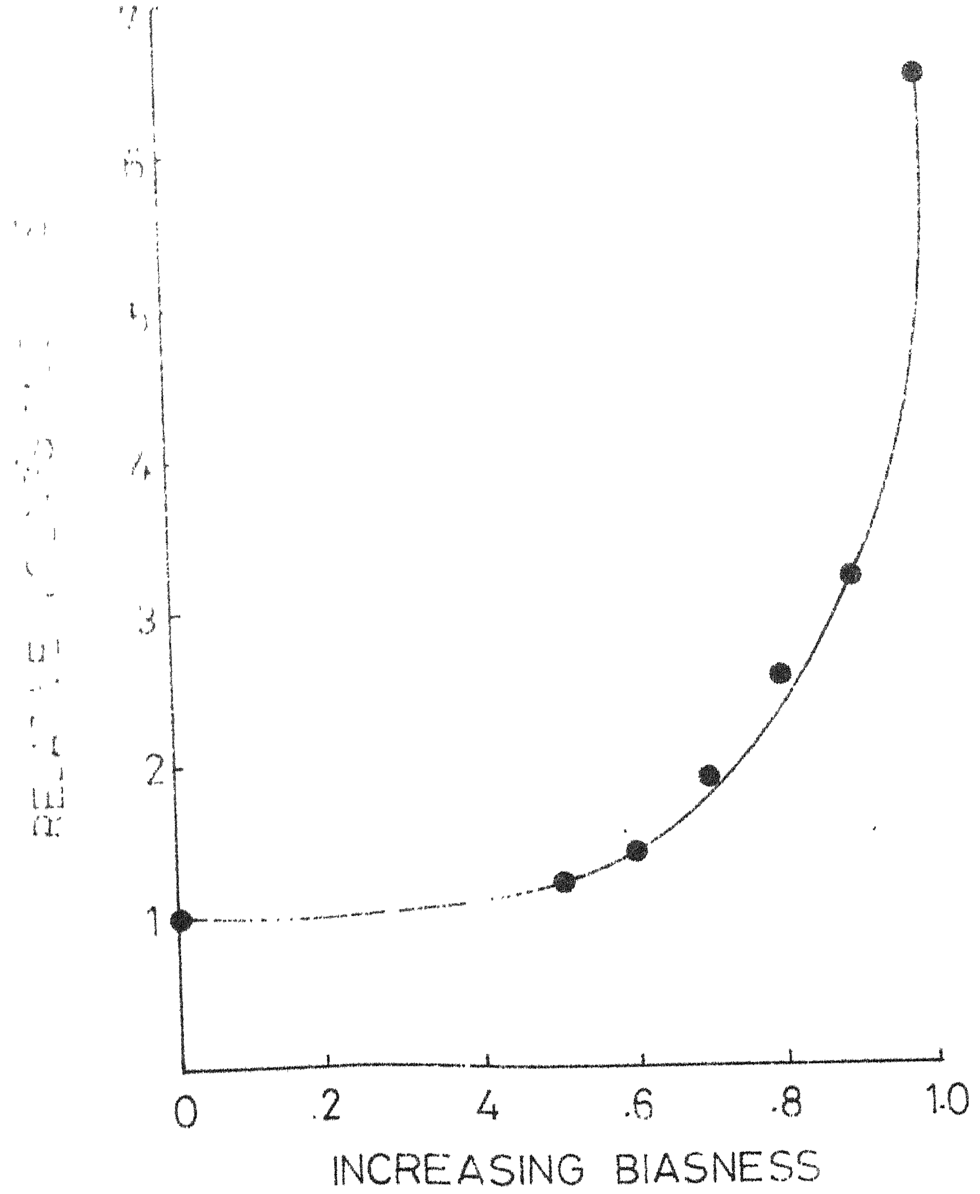


FIG. 25- EFFECT OF BIASNESS ON YOUNG'S MODULUS ($V_f = 0.3$)

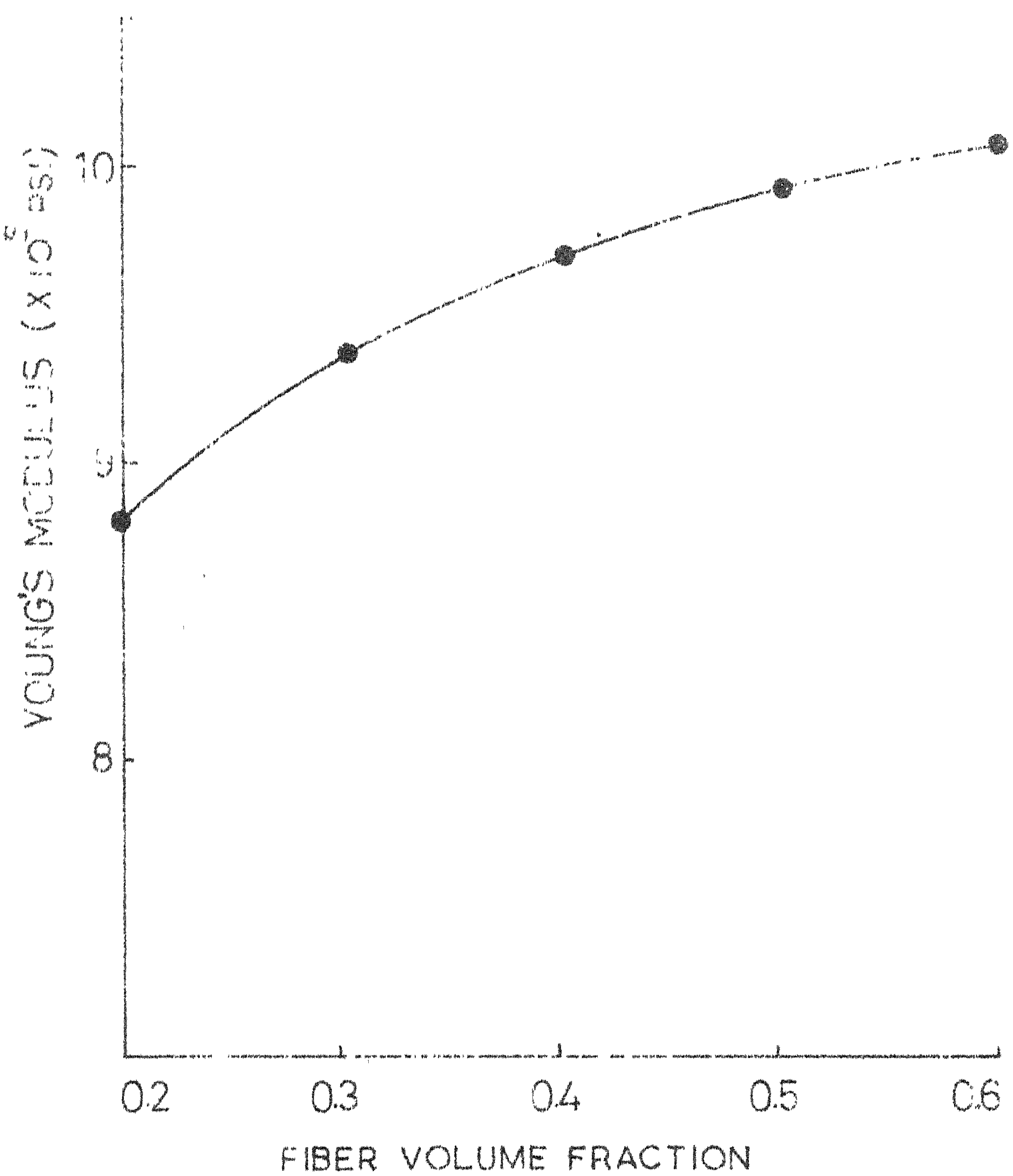


FIG. 26 EFFECT OF FIBER VOLUME FRACTION ON YOUNG'S MODULUS FOR RANDOM FIBERS.(GLASS EPOXY SYSTEM)

TABLE I

COMPARISON OF DISPLACEMENTS AT SELECTED NODES

Case	No. of nodes	No. of elements	Displacements in inches (x 10^{-4})				Reference	Formulation
			Point A (4,5, 3.0)		Point B (2.25, 1.5)			
			u	v	u	v		
I	9	4	1.361444	-1.161279	0.3182607	-0.2936801	Self	CST
II	25	16	1.455171	-1.239908	0.3679722	-0.3123025	Desai-Abel	CST
III	81	64	1.486224	-1.264569	0.3732400	-0.3171139	Desai-Abel	CST
IV	9	4	1.500070	-1.275024	0.3750026	-0.3187633	Self	Hybrid (5 terms)
V	-	-	1.500000	-1.275000	0.3750000	-0.3187500	--	Exact

TABLE II

COMPARISON OF STRESSES AT SELECTED POINTS

Number of Nodes: 9 ;   Number of Elements: 4.

Case	Stresses in psi						Reference	Formulation
	Point D (1.125, 0.75)			Point E (3.375, 2.25)				
	σ_x	σ_y	σ_{xy}	σ_x	σ_y	σ_{xy}		
I	231.09	-3.6239	2.0463	691.06	2.9185	-11.324	Self	CST
II	250.00	-4.1046×10^{-3}	-4.7579×10^{-3}	749.99	2.6855×10^{-3}	-1.5564×10^{-3}	Self	Hybrid (5 terms)
III	250.00	0	0	750.00	0	0	---	Exact

TABLE III

COMPARISON OF ELASTIC PROPERTIES CALCULATED BY FINITE ELEMENT METHODS

Methods	Average Stress in psi	Average Longitudinal Strain	Average Transverse Reduction	Young's Modulus psi	Poisson's Ratio
Displacement	8000.00	0.0154323	0.005969	5.18×10^5	0.39
Hybrid	8000.00	0.01545282	0.006198	5.177×10^5	0.40
Exact	--	--	--	5.0×10^5	0.35

TABLE IV

COMPARISON OF LONGITUDINAL AND TRANSVERSE MODULI CALCULATED USING DIFFERENT METHODS

	Fiber Volume Fraction	Finite Element Methods		Paul's Results Ref. 27	Rath, Sinha, Rao Ref. 28	Greszczuk Ref. 29
		Pure Displacement Method	Mixed Element Method			
Longitudinal Young's Modulus	0.6	6.9	6.9	6.6	4.5	6.6
Transverse Young's Modulus	0.3	0.56	0.56	0.70	0.70	0.86

TABLE V

COMPARISON OF ELASTIC PROPERTIES OF RANDOMLY ORIENTED FILAMENTARY COMPOSITE CALCULATED USING DIFFERENT METHODS

	Young's Modulus (psi)	Poisson's Ratio	Shear Modulus (psi)
Present Analysis	0.94×10^6	0.146	0.41×10^6
Tsai and Pagano	1.28×10^6	--	0.475×10^6
Krenchel	0.9975×10^6	0.335	0.45×10^6
Christensen and Waals	1.62×10^6	--	--

TABLE VI

EFFECT OF COMPRESSIVE INITIAL STRESS ON THE YOUNG'S MODULUS OF RANDOMLY ORIENTED FILAMENTARY COMPOSITES

		Compressive Initial Stresses in psi				
		NIL	250	750	1000	2000
Maximum stress normalised with respect to the average applied tensile stress	σ_y (tensile)	4.51	4.48	4.42	4.38	4.26
	σ_x (compressive)	1.45	1.49	1.54	1.56	1.67
	σ_{xy}	0.41	0.44	0.49	0.52	0.66
Young's modulus x 10^5 psi		9.38	9.39	9.41	9.42	9.45